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An electro-optic modulation technique for direct and accurate measurement of birefringence

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Abstract

We report a convenient and accurate technique for measuring the optical path difference induced by a birefringent crystal. The method is based on the transmission of a low-frequency signal by the electro-optic amplitude modulation of a monochromatic light source. The linear superposition principle of the induced birefringence and the electro-optic effect are employed in the analysis of the optical parameters. Birefringence measurements at the wavelength 0.6328 μm for some common nematic liquid crystal cells are presented. The measurement accuracy is close to 10^{-3}. In addition, the phase shift of a twisted-nematic liquid crystal spatial light modulator, which is now widely used in optical processing systems, is determined as a function of the gray-scale level.

1. Introduction

The precise determination of the phase retardation, and thereby of the birefringence for certain materials (liquid crystal, for example) plays an important role in various fields of research. Photonic switching operation in free space, for example, requires devices that rely on the separation of polarization or the deflection of light beams by means of liquid crystal cells whose birefringence is controlled electrically. The accurate measurement of their optical path difference is useful to optimize such structures.

These optical parameters can be measured by monitoring the wave numbers at which maxima and minima occur in the transmittance of the optical system where the sample to be tested is placed between crossed or parallel polarizers [1,2]. Such methods are relatively simple, but they present major drawbacks such as a nonlinear relationship between birefringence and the transmitted intensities, and only modest precision.

Several interferometric techniques have also been developed for measuring the index of refraction and/or the birefringence of anisotropic samples. These techniques use typically a

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Mach–Zehnder or a Michelson interferometer [3] in which the optical path difference of a light beam is modified by the presence of the sample under investigation. Because of its sensitivity, the measurement is affected by external perturbations such as mechanical vibrations. Interferometric phase modulation technique [4] can remedy these difficulties to measure retardations with a high sensitivity and precision but to the expense of complexity of the technique. Also a polarization ring interferometer has been used for measuring the induced birefringence arising from the electro-optic response of film samples [5]. More recently, an optical imaging system combined with different digital post-processings has been proposed for optical path difference measurements of electrically addressed spatial light modulators [6].

We describe in this paper a novel layout of a single light-beam optical interferometer suitable for measuring the optical path difference of anisotropic materials that alleviate all above limitations. Our architecture, whose implementation is relatively simple, is based on the electro-optic modulation by Pockels effect of low-frequency signals.

The paper is divided into three main sections. In Section 2, the basic architecture and the principle of operation are presented. Experimental results of phase difference measurements and birefringence determinations at the wavelength 0.6328 μm for nematic liquid crystal cells are given in Section 3. Finally, we demonstrate that this technique can be used to assess phase-shift measurements for liquid crystal displays which can be used as spatial light modulators.

2. Description of the system and principle of operation

Fig. 1 shows how the method can be used in monochromatic electro-optic transmission system for measuring the optical path difference (OPD) of anisotropic materials. The optical configuration consists of an emitting module driven by a low-frequency signal $f(t)$, a birefringent sample Q to be tested and a classical photo-detector (PD). The whole system is illuminated by a monochromatic light beam. The emitting module is formed by a bulk transverse field electro-optic modulator (EOM) (employing four identical 45° X-cut ADP ($NH_4H_2PO_4$) crystals oriented at 90° with respect to each other to cancel ADP natural birefringence), in series with a quarter-wave plate (QWP) set between crossed polarizer P and analyzer A. The fast and slow axes of the plates and of the modulator are parallel to each other, but are oriented at an angle of 45° to the polarization directions of P and A. The birefringent sample Q exhibits an optical delay $D_o = e\delta n$ between its fast and slow axes (e is the sample thickness, and δn its birefringence). Under these conditions, the emitting module works as a polarization interferometer with a path imbalance $\lambda/4 + D_o$, and a theoretical loss of −3 dB.

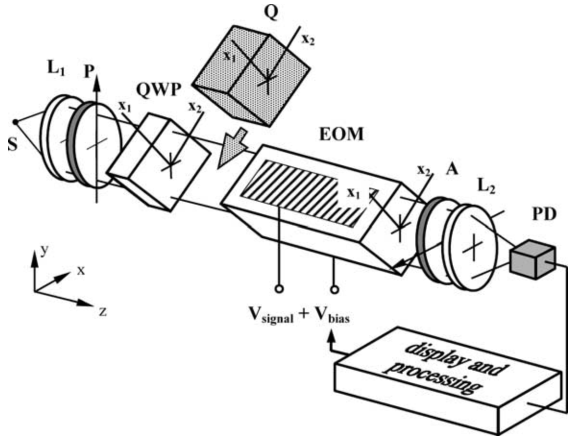

Fig. 1. Experimental set-up for phase-shift measurement: S, He–Ne laser; L_1, L_2, focusing lenses; P, A linear polarizers; QWP, quarter-wave plate; EOM, electro-optic modulator; PD, photodetector.

Let E_0 be the amplitude of the wave incident on the crystal face. When passing through QWP + EOM, the incident light beam, which is linearly polarized say at 45° to the fast and slow axes, is split in two orthogonal emergent components $\vec{E}_1$ and $\vec{E}_2$ such as

$$\vec{E}_1 = \frac{E_0}{\sqrt{2}} \sin(\omega t) \cdot \vec{x}_1; \qquad \vec{E}_2 = \frac{E_0}{\sqrt{2}} \sin(\omega t + \Gamma) \cdot \vec{x}_2, \tag{1}$$

Γ being the phase-difference between the ordinary and extraordinary components.

A simple JONES calculus [7] gives the intensity of the light transmitted by the analyzer A crossed to the polarizer P. The intensity I of the detected beam is then given by

$$I_\perp = I_0 \sin^2 \frac{\Gamma}{2} \quad \text{or} \quad I_\perp = \frac{I_0}{2}(1 - \cos \Gamma), \tag{2}$$

where I_0 is the light intensity under full transmission.

The intensity of the light transmitted by the analyzer undergoes a modulation that depends on the phase-difference and thus on the birefringence introduced by the EOM. Optical transmission based on intensity electro-optic modulation constitutes a direct application of relation (2) between the light intensity transmitted by an electro-optic modulator placed between parallel or crossed polarizers and the phase-difference Γ introduced by the modulator. The latter works as a two-wave interferometer.

The emitting module encodes the signal $f(t)$ as an optical phase-difference Γ given by

$$\Gamma = \frac{2\pi}{\lambda_0} K V_{\text{signal}} + \frac{\pi}{2}, \tag{3}$$

where

$$K_{(\mu\text{m/V})} = \frac{4 r_{41} n_0^3 n_e^3 \sqrt{2} L}{(n_0^3 + n_{e^3})^{3/2} d}$$

is a constant attached to EOM, r_{41} is a Pockels coefficient, n_o and n_e are the ordinary and extraordinary refractive indices of the electro-optic crystal, L and d are its length and thickness, respectively, λ_0 is the wavelength of the incident light beam and V_{signal} is the driving voltage. The applied voltage thus inserts adjustable phase-difference Γ between the two field components. The $\pi/2$ phase retardation is introduced by the QWP whose eigen axes coincide with those of the EOM in the presence of the applied electric field. Expression (2) then becomes

$$I = \frac{I_0}{2}\left(1 + \sin \Gamma'\right), \tag{4}$$

where $\Gamma' = (2\pi/\lambda_0) K V_{\text{signal}}$ is the phase change introduced by the electro-optic crystal. Its operating conditions are such that $\Gamma' \ll 1$ rad. Eq. (4) then reduces to

$$I = \frac{I_0}{2}\left(1 + \frac{2\pi}{\lambda_0} K V_{\text{signal}}\right). \tag{5}$$

The intensity transmitted by the emission system varies linearly with the electric field applied to the modulator. This linearity is achieved by inserting a quarter-wave plate, or by applying a bias voltage $V_{\text{bias}} = V_{\pi/2}$, which plays the role of a continuous component [8]. Fig. 2 illustrates the situation when the transmission is carried out without continuous component (nonlinear response) and with continuous component (linear response). Then, for linear operation, the EOM is biased near the point F.

It is this principle, also called *compensation method*, that is exploited to obtain the additional phase induced by a birefringent material. While inserting in the previous optical set-up (see Fig. 1) the object Q to be characterized, the phase difference produced by the unit (QWP + Q + EOM) is then written

$$\Gamma = \frac{2\pi}{\lambda_0} K V_{\text{signal}} + \frac{\pi}{2} + \Gamma_Q, \tag{6}$$

where Γ_Q is the phase retardation introduced by the cell to be tested.

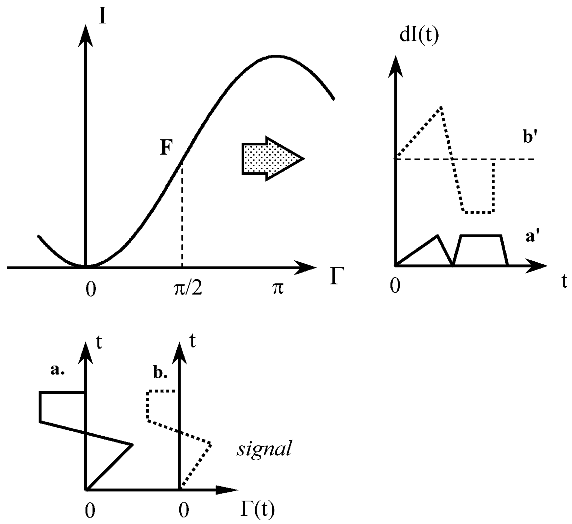

Fig. 2. Phase retardation dependence of the transmitted light intensity of an electro-optic modulator set between crossed polarizer and analyzer: (a) without quarter-wave plate, nonlinear response (solid line, a′); (b) with a quarter-wave plate, about the operating point F, the response is linear (dotted line, b′).

In this case, the linearity at the detection is not guaranteed any more. However, if we wish to obtain once again a linear variation between the detected intensity I and the driving voltage V_{signal}, it is necessary to compensate the phase delay by means of a well-known constant bias voltage V_{bias}. In this way, we will operate about the operating point F (see Fig. 2). The additional bias voltage, also called *compensation voltage*, introduced by the modulator will induce an optical path difference and thus a phase retardation related to the object to be tested, such as

$$\Gamma_Q = \frac{2\pi}{\lambda_0} K V_{\text{bias}}. \tag{7}$$

In the following, we will detail the various stages which will allow to measure, with high accuracy, the birefringence of nematic liquid crystal materials and the phase retardation introduced by a twisted-nematic liquid crystal display.

3. Experiments and results

3.1. Preliminary experiments

In order to test the validity of the method and assess the ease of its implementation, the preliminary experiments have been performed with liquid crystal (LC) cells in nematic phase. To understand the experimental results, let us briefly recall the physics of the nematic liquid crystal.

A nematic liquid-crystal cell consists of a thin nematic LC layer placed between two parallel glass plates that are coated with thin layers of transparent conductive material and rubbed so that the molecules are parallel to each other. The material then acts as a uniaxial crystal with the optical axis parallel to the molecule orientation. We use a Cartesian coordinate system with its z-axis perpendicular to the LC layer. The fast axis of the LC molecule, so called "*active molecular director*" specified by the vector $\boldsymbol{u}$, is oriented at an angle of 45° with respect to the y-axis (Fig. 3(a)). When a polarized monochromatic light propagates in the z direction through the LC cell with its polarization axis along the y-axis of the Cartesian coordinate system, a cell of thickness e provides a phase retardation [9]

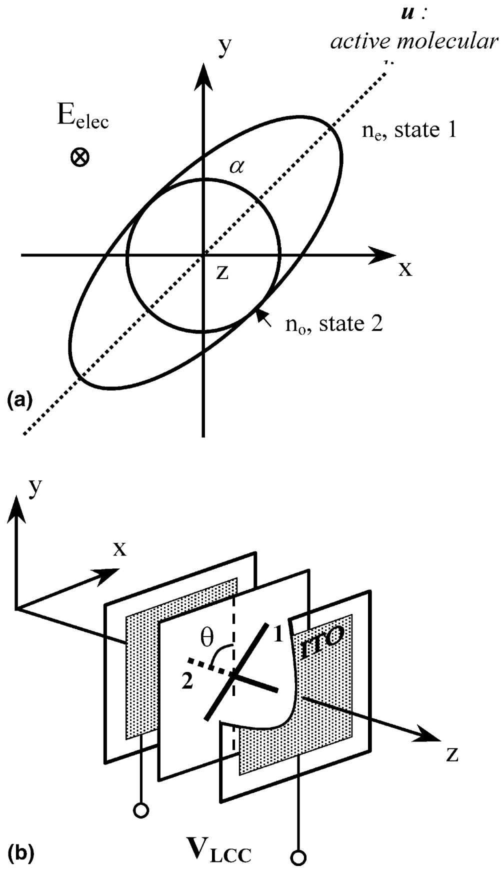

Fig. 3. Nematic liquid crystal material treated as uniaxial material: (a) the molecule of LC rotate and align with the applied electric filed (from state 1 to state 2); (b) configuration of the nematic LC cell with ITO (indium tin oxide) transparent electrodes.

$$\Gamma_{\text{LCC}} = \frac{2\pi}{\lambda_0} \delta n e \sin^2 \theta, \tag{8}$$

where θ is the angle between the optical axis (specified by the vector $\boldsymbol{u}$) and the light propagation direction (Fig. 3(b)). Generally the pre-tilt angle $(90° - \theta)$ of the LC is small (~2°), and Eq. (8) reduces to

$$\Gamma_{\text{LCC}} = \frac{2\pi}{\lambda_0} \delta n e. \tag{9}$$

The "local" birefringence δn of the LC varies as a function of the electric field $\vec{E}_{\text{elec}}$ applied in the z-direction. When no voltage is applied, the molecule director is parallel to the glass surfaces. If an electric field $\vec{E}_{\text{elec}}$ is applied in the z-direction (by applying a voltage across transparent conductive electrode coated on the inside of the glass plates), the molecules tend to align in the direction of the electric field. When the latter is sufficiently large, the tilt of the LC molecules approaches 90°, and the LC becomes an isotropic layer with a single refractive index n_o. The tilting of the molecules induces a reduction of the extraordinary refractive index so that [10]

$$\frac{1}{n^2(\theta)} = \frac{\cos^2\theta}{n_e^2} + \frac{\sin^2\theta}{n_o^2}, \tag{10}$$

where n_o (in state 2: $V_{LCC} = V_{max}$) is the ordinary refractive index and n_e is the off-state extraordinary refractive index (in state 1: $V_{LCC} = 0$). Thus the phase retardation is reduced to

$$\Gamma_{LCC} = \frac{2\pi}{\lambda_0}[n(\theta) - n_o]e. \tag{11}$$

The tilt angle and thus the phase-shift introduced by the LC cell is related to the applied voltage.

The experiments were made on a ~7 μm-thick nematic liquid crystal (E. Merck Chemicals, Germany) cell (RC052, from France Telecom Research & Development) and on a ~3 μm-thick nematic liquid crystal cell (C1282, from Niotik, Moscow). Indium tin oxide (ITO) thin films were used as transparent electrodes. The He–Ne laser ($\lambda = 632.8$ nm) was employed for phase-shift measurements of the LC sample. The overall measurement process is detailed below:

(1) In order to check that the detected signal arises from the Pockels effect, transmission by amplitude electro-optic modulation of 1 MHz analog signal was carried out through a He–Ne laser light-channel at 632.8 nm which is linearly polarized at an angle of 45° with respect to the fast and slow axes of the combination (QWP + EOM). The modulated intensity varies linearly with the driving signal. The bias voltage V_{bias} was monitored by an oscilloscope whereas the signal at the photodiode detector output was controlled by an electrical spectrum analyzer.

(2) The liquid crystal cell was placed in the optical system as illustrated in Fig. 1. Beforehand, we would have determined by means of a polarized two-wave interferometer (Michelson, for example) the dynamic axis of the liquid crystal molecule. The sample to be tested was adjusted so that its fast and slow axes were being parallel to those of the QWP and EOM. An ac voltage V_{LCC} applied to the LC cell was controlled by an electric wave generator connected to a personal computer. The frequency and the waveform of the applied voltage were 10 kHz and bipolar rectangular, respectively. The LC cell introduces a phase retardation given by $\Gamma_{LCC} = 2\pi e[n_e(V_{LCC}) - n_o]/\lambda_0$. The linearity at the detection is not realized any more.

(3) V_{bias} was adjusted until obtaining again a linear variation between the detected intensity and the driving signal. Our technique of birefringent measurement relies on the suppression, on the spectrum analyzer, of the high order harmonics of the rf signal, with excellent signal to noise ratios. The phase-shift induced by the sample is estimated from Eq. (7).

(4) The variations of the birefringence as a function of the V_{LCC} was calculated with, $\delta n_{LCC} = \Gamma_{LCC}\lambda_0/2\pi e$.

For the used bulk ADP transverse electro-optic modulator, $L = 8 \times 10^{-2}$ m, $d = 2.5 \times 10^{-3}$ m, $r_{41} = 2.45 \times 10^{-11}$ m/V, $n_o = 1.523$ and $n_e = 1.478$, illuminated by a He–Ne laser ($\lambda_o = 632.8$ nm), Eq. (7) becomes

$$\Gamma_{bias} = 0.01686\pi V_{bias} \quad (\text{modulo } 2m\pi) = \Gamma_{LCC}, \tag{12}$$

where m is an integer and V_{bias} is the continuous voltage applied to the EOM which plays the role of the compensation voltage. The voltage-dependence birefringence of the LC cell is then given by,

$$\delta n_{LCC}(V_{LCC}) = \frac{\lambda_0[2m + 0.01686 V_{bias}(V_{LCC})]}{2e}. \tag{13}$$

An optical-path difference shorter than the optical wavelength can be directly deduced from Eqs. (7) and (9). When the optical retardation is larger than the wavelength, ambiguities of an integer multiple of 2π occur in the optical-path difference measurement. To avoid these ambiguities, inherent to monochromatic interferometry, it is necessary to measure the traditional optical transmission of polarizer – LC cell – analyzer system as a function

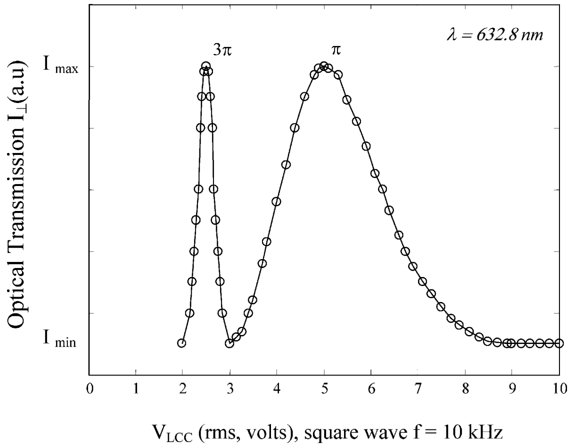

Fig. 4. Optical transmission versus of the voltage for 7 μm-thick nematic LC (from E. Merck Chemicals, Germany) cell, placed between crossed polarizer and analyzer, at $\lambda = 632.8$ nm.

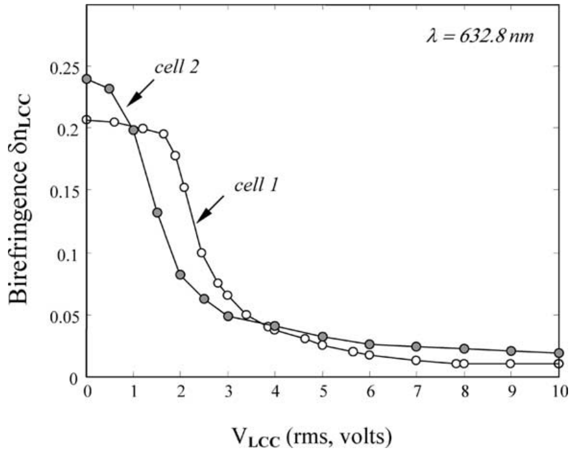

Fig. 5. Voltage dependence of the birefringence of nematic liquid crystal for cell 1 (~7 μm-thick nematic LC from E. Merck Chemicals, Germany) and cell 2 (~3 μm-thick nematic LC from Niotik, Moscow); $\lambda = 632.8$ nm.

of the amplitude of the applied signal V_{LCC}. Fig. 4 shows the experimental result corresponding to crossed polarizer and analyzer at the 632.8 nm He–Ne laser wavelength. We see that the $I_{\perp}$ goes through maxima and minima as the amplitude of the applied signal increases and that it approaches limiting values in the high voltage region. The presence of two maxima indicates that the variation of optical-path difference is λ, then $m = 1$. The previous equation becomes,

$$\delta n_{\mathrm{LCC}}(V_{\mathrm{LCC}}) = \frac{\lambda_0[2 + 0.01686 V_{\mathrm{bias}}(V_{\mathrm{LCC}})]}{2e}. \quad (14)$$

Fig. 5 shows the dependence of LC effective birefringence on voltage at $\lambda = 632.8$ nm for the LC cells used. The birefringence decreases linearly as the voltage exceeds the Freedericksz transition [11] and the rate of decrease is less in the high voltage region. We obtain $\delta n_{\mathrm{cell1}} = 0.206$ for the RC052 liquid crystal cell and $\delta n_{\mathrm{cell2}} = 0.241$ for the C1282 liquid crystal cell. The measuring accuracy is close to 10^{-3}.

3.2. Phase-shift measurements of liquid crystal SLM

In this section, we show how the optical measurement technique described in Section 2 can be used to provide the phase difference or the birefringence of the twisted-nematic liquid crystal displays (LCDs). The liquid crystal is usually sandwiched between a polarizer and an analyzer. The phase properties of such material are determined by a number of parameters, such as the applied voltage level and the input and the output polarizers angles. Our current interest in these devices lies in their use as electrically addressed spatial light modulators (EASLMs) in an optical correlator, both in the input and filter planes of the VanderLugt architecture, for example[12]. The optimum filters are implemented numerically, which generally involves a 2-D Fourier transform algorithm to compute the complex-valued filter [13].

To use these LCDs as programmable SLMs, the anisotropic parameters, such as the twist angle and the birefringence of the liquid crystal must be measured [14,15]. The twisted nematic liquid crystal has the optical properties of a uniaxial birefringent material whose optical axis is parallel to the direction of the molecule. When no voltage is applied, the orientation of the extraordinary axes of the LC molecules is parallel to the substrate orientation and forms a helicoidal structure. When voltage is applied to LCD perpendicular to its surface, the molecules tend to align in the direction of the induced electric field. This yields a modification of the LCD birefringence. The intensity transmittance and the phase shift are functions of only one variable, the birefringence which depends on the applied voltage and hence on the gray-scale level. The polarizers permit to control the amplitude and the phase of the emergent light, which have in general a mutual dependence.

Our experiments were performed with a CRL (VGA1) active matrix liquid crystal spatial light modulator having 640 (horizontal) $\times$ 480 (vertical) pixels of pitch 24 μm and 31 μm, respectively.

The experimental set-up is that shown in Fig. 1. The LCD to be characterized was placed between the quarter-wave plate and the electro-optic modulator so that its birefringent axes were parallel to those of QWP and EOM. An uniform image with a given gray level was written onto the EASLM where every pixel is addressed with the same voltage. Phase shift was measured as a function of the gray-scale level. The results for He–Ne laser ($\lambda = 632.8$ nm) are shown in Fig. 6. The maximum phase shift is about 280°. This experimental result, which is a good agreement with those previously published by some authors [16], shows that the amplitude and phase modulations are coupled. To use the device as an amplitude-only or phase-only modulator, we must identify a configuration (i.e, the orientation of polarizers) and an interval of values of the birefringence (or the phase shift) for which one modulation variable does not affect the other. For gray level larger than 50, the phase modulation is almost linear function of the gray level (see Fig. 6). The intensity modulation does not much affect the phase modulation.

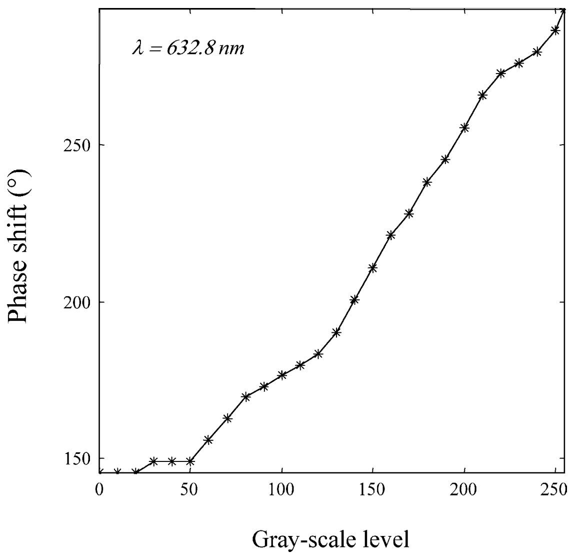

Fig. 6. Phase-shift versus the gray-scale level, for He–Ne laser illumination ($\lambda = 632.8$ nm)

This optical characterization is necessary to determine the SLM operating curve [17] which is defined as the covariation of amplitude and/or phase as a function of the driving voltage. This operating curve allows the implementation of the complex-value filter on the modulator dedicated to the optical correlation (Knowledge of the exact operating curves of the input and filter-modulators allows optimum filters to be calculated for the correlator).

4. Conclusions

We have reported a new technique for measuring the birefringence in film samples. The method, very simple experimentally, is based on the linear variations of the transmitted intensity with the applied electric field to an amplitude modulator. The method was demonstrated using thin nematic LC plates and twisted-nematic LCD. The measurement scheme is shown to provide a birefringence with accuracy of the order of 10^{-3} for the nematic LC samples. This high-stability phase measurement scheme can be used to measure the dependence of the birefringence on wavelength, yielding an approximate estimate of the birefringence dispersion. Furthermore, the same optical architecture has been used to measure the dependence of LCD phase-shift on applied voltage and hence on gray-scale level. Knowledge of this optical parameter of LCDs which are used as an input and/or filter modulators allows to design optimum filters to optimize the correlation function.

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