Department of Electrical and Computer Engineering
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OPTICS LAB -ECEN 5606

Experiment No. 13

ACOUSTOOPTIC DEVICES

I. Introduction

In this lab you will learn to use acoustooptic modulators and deflectors in order to manipulate the intensity, frequency, and direction of propagation of laser wavefronts. You will optimize the operation of an acoustooptic modulator for large diffraction efficiency, and wide modulation bandwidth, and you will learn the difference between modulators and deflectors.

II. Background

Acoustooptic devices are based on the periodic modulation of the optical index of refraction caused by an acoustic wave propagating in a transparent material. An optical wave passing through the region containing the acoustic wave will experience a periodic phase modulation that can produce a corrugation of the optical wavefront which will result in a scattering of the optical wave into diffracted orders. The acousto-optic interaction is somewhat different from other types of optical volume diffraction effects because the perturbation is required to be a superposition of propagating eigenmodes of the acoustic wave equation. This is especially relevant in the case of highly anisotropic acoustics, such as in the slow shear mode of paratellurite TeO_2.

The acousto-optic interaction results in the phase modulation of an optical beam by an acoustic wave in a photoelastic medium. In order to properly describe this interaction the acoustic and optical waves will be represented separately, and then the coupling between them will be introduced. This interaction is mediated by the 4th rank photoelastic tensor which describes the dielectric impermeability tensor perturbation caused by the propagating strain wave. In practice, devices are designed to make use of only a single element of the coupling tensor, and the problem is greatly simplified.

II.1 Acoustic eigenmodes

The acoustic wave is modelled as a time and space varying particle displacement vector field $\mathbf{u}(\mathbf{x},t)$, which at each crystal lattice site describes the particle displacement from its equilibrium position [2,3]. A sinusoidal displacement wave of amplitude W, radian frequency Ω, wave vector $|\mathbf{K}|=2\pi/\Lambda$, acoustic wavelength Λ, propagating with a phase velocity $v_a(\mathbf{s})=\Omega/|K|$ in the direction defined by the unit vector $\mathbf{s}=\mathbf{K}/|K|$, and with unit polarization $\hat{U}$ is described by the particle displacement field

$$\mathbf{u}(\mathbf{x},t) = W\hat{U}\cos(\Omega t-\mathbf{K}\cdot\mathbf{x}) = W\hat{U}\cos\left[\Omega(t - \frac{\mathbf{s}\cdot\mathbf{x}}{v_a(\mathbf{s})})\right] \tag{1}$$

The displacement gradient matrix is the spatial derivative of the displacement field, and its components are given by

$$Q_{ij}(\mathbf{x},t) = \left[\frac{\partial u_i(\mathbf{x},t)}{\partial x_j}\right] \tag{2}$$

The symmetric part of the displacement gradient matrix is known as the linearized Strain tensor and its components are given by

$$\mathbf{S}_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] \tag{3}$$

The Stress tensor $\mathbf{T}_{ij}$, which is symmetric in non feroic materials, is related to the Strain tensor through the 4th rank elastic stiffness tensor in a generalization of Hooke's law.

$$\mathbf{T}_{ij}(\mathbf{x},t) = \mathbf{c}_{ijkl}\mathbf{S}_{kl} \tag{4}$$

Where the Einstein summation convention over repeated indices is implied, and i,j,k,l may take on any of the three spatial directions x_1, x_2, x_3, or equivalently x,y,z.. The elastic coefficients possess certain symmetries because of the symmetry of $\mathbf{S}$ and $\mathbf{T}$, so that $\mathbf{c}_{ijkl} = \mathbf{c}_{jikl} = \mathbf{c}_{ijlk} = \mathbf{c}_{jilk}$, and energy arguments show that $\mathbf{c}_{ijkl} = \mathbf{c}_{klij}$. The acoustic field equations show that energy oscillates between the stress energy and the strain energy in a fashion that is analogous to the electromagnetic oscillation between electric and magnetic energy. The dynamical equation of motion for a vibrating medium relates the restoring force as given by the divergence of the Stress tensor with the mass times acceleration of the displacement field.

$$\vec{F} = \nabla \bullet T = \rho_m \frac{\partial^2 \vec{u}}{\partial t^2} \tag{5a}$$

$$F_i = \frac{\partial T_{ij}}{\partial x_j} = \rho_m \frac{\partial^2 u_i}{\partial t^2} \tag{5b}$$

In this equation ρ_m is the scalar equilibrium mass density of the medium. Substitution of the Stress-Strain relationship, and the definition of Stress in terms of particle displacements into the dynamical equation of motion results in the differential equation governing the propagation of particle displacement fields.

$$F_i = \mathbf{c}_{ijkl} \frac{\partial^2 u_k}{\kappa x_l \kappa x_j} = \rho_m \frac{\kappa^2 u_i}{\kappa t^2} \tag{6}$$

Substitution of the assumed plane wave of Equation 1 into this equation results in the equation for the allowed modes of propagation.

$$\mathbf{c}_{ijkl} s_j s_l k^2 U_k = \Gamma_{ik}(\vec{s}) k^2 U_k = \rho_m \Omega^2 U_i \tag{7a}$$

$$\left[\mathbf{c}_{ijkl} s_j s_l / v_a^2 - \rho_m \delta_{ik} \right] U_k = 0 \tag{7b}$$

This system is only compatible for all waves if the determinant of the system is zero, which results in the dispersion relationship in terms of the Christoffel matrix $\Gamma_{ik}(\mathbf{s}) = \mathbf{c}_{ijkl}\ s_j s_l$ as a function of the propagation direction $\mathbf{s}$, where s_x, s_y, s_z are the appropriate direction cosines.

$$\det \left| \frac{\Gamma_{ik}(\vec{s})}{v_a^2} - \rho_m \delta_{\iota\kappa} \right| = 0 \tag{8}$$

This equation has three solutions for the acoustic slowness, or inverse velocity $1/v_a(\mathbf{s}) = \mathbf{K}/\Omega$ for each direction $\mathbf{s}$, forming three equivalent frequency scaled surfaces in $\mathbf{K}$ space, known as acoustic momentum space. The corresponding eigen-polarizations $\hat{U}(\mathbf{s})$ for each direction of propagation correspond to the longitudinal (or quasilongitudinal) and two shear (or quasishear) solutions.

An example of cross sections through this surface, calculated via an explicit solution of the Christoffel equation are shown in Figure 1 for the important case of paratellurite TeO_2[4]. The anomalously slow shear mode is recognized by the large lobes extending in the [110] direction. The acoustic velocity along this axis is .62mm/μsec, and the radius of curvature in the x-y plane is about 1/44, and in the xy-z plane that orthogonally cuts through the slow shear lobe the radius of curvature is about 1/12. The acoustic phase velocity surface is found by inverting the slowness surface radially about the unit sphere. It is important to realize that the acoustic Poynting's vector, which describes the direction of energy flow, is orthogonal to the slowness surface, thus in the regions of high curvature around the [110] direction slightly off axis acoustic components will not only propagate faster, they also will rapidly walk off from the main beam.

II.2 Optical Eigenmodes

Optical propagation through a homogeneous, lossless anisotropic medium can be described in terms of Maxwell's equations [5]. Faraday's law gives the relation between an induced electric field and a time varying magnetic field. Ampere's law describes the creation of a magnetic field due to a dielectric flux, a conductivity current, and a source current, however for the case of interest to acousto-optics, no currents are present. Similarly, we will assume no free electric charges, and of course no free magnetic monopoles.

$$\nabla \times \vec{E} = -\frac{\kappa \vec{B}}{\kappa t} \tag{9}$$

$$\nabla \times \vec{H} = \frac{\kappa \vec{D}}{\kappa t} + \vec{J}_c + \vec{J}_s = \frac{\kappa \vec{D}}{\kappa t} \tag{10}$$

$$\nabla \Box \vec{D} = \rho_e = 0 \tag{11}$$

$$\nabla \Box \vec{B} = 0 \tag{12}$$

In an optically anisotropic medium the displacement vector **D** and electric field **E** are not necessarily parallel, and are related by the Hermitian second rank permittivity tensor ε. In magnetically isotropic material the magnetic vector **H** is related to the magnetic induction **B** by the scalar permeability μ. The resulting constitutive relationships describe the effect of material media on the propagation of electromagnetic waves, and allows the unique solution to Maxwell's equations with a given set of boundary conditions.

$$\vec{D} = \varepsilon \vec{E} = \varepsilon_0 \vec{E} + \vec{P} \tag{13a}$$

$$D_i = \varepsilon_{ij} E_j = \varepsilon_0 (\delta_{ij} + \chi_{ij}) E_j = \varepsilon_0 E_i + \chi_{ij} E_j = \varepsilon_0 E_i + P_i \tag{13b}$$

$$\vec{B} = \mu \vec{H} = \mu_0 \vec{H} + \vec{M} \tag{14a}$$

$$B_i = \mu H_i \tag{14b}$$

The permittivity tensor is expressed as the free space permittivity ε_0 plus a material dependent susceptibility tensor, $\varepsilon_{ij}=\varepsilon_0(1+\chi_{ij})$, and the presence of the matter is seen to induce a polarization vector **P** which is related to **E** through the linear susceptibility tensor χ_{ij}.

In order to derive the optical eigenmodes we will assume an electromagnetic plane wave with angular frequency ω, and wave vector $|\mathbf{k}|=2\pi/\lambda$, propagating in the direction of the unit vector $\mathbf{s}=\mathbf{k}/|\mathbf{k}|$, with a phase velocity $v_p=c/n=1/(\mu\varepsilon)^{1/2}$. The refractive index $n=(\varepsilon/\varepsilon_0)^{1/2}$ is a function of the direction of propagation, the polarization of the wave, and the frequency if the material is dispersive, and it is the allowed eigen-velocities v_p and eigen-polarizations $\mathbf{E}_0$ which are to be determined.

$$\vec{E}(\vec{x},t) = \vec{E}_0 e^{-i(\omega t-\vec{k}\cdot\vec{x})} \tag{15a}$$
$$\vec{H}(\vec{x},t) = \vec{H}_0 e^{-i(\omega t-\vec{k}\cdot\vec{x})} \tag{15b}$$

We can now substitute these assumed solutions into Maxwell's equations to obtain

$$\vec{k} \times \vec{E} = \omega\mu\vec{H} \tag{16a}$$
$$\vec{k} \times \vec{H} = -\omega\varepsilon\vec{E} \tag{16b}$$

Substituting the first equation for **H** into the second equation yields an equation for **E**.

$$\vec{k} \times (\vec{k} \times \vec{E}) + \omega^2\mu\varepsilon\vec{E} = 0 \tag{17a}$$
$$\left[k_i k_j - \delta_{ij}k^2 + \omega\mu\varepsilon_{ij}\right] E_j = 0 \tag{17b}$$

In the absence of optical activity, the symmetry of the permittivity tensor ε allows us to rotate to a principal dielectric coordinate system where ε_{ij} is purely diagonal. In order for a nontrivial solution to exist the determinant of the matrix in brackets in equation (17b) must be zero.

$$\det[k_i k_j - \delta_{ij}k^2 + \omega\mu\varepsilon_{ij}] = 0 \tag{18}$$

This is the equation for the optical normal surface in **k** space, referred to as optical momentum space. It is analogous to the acoustic momentum slowness surface, and the inverse optical phase velocity $v_p^{-1}=n/c=k/\omega$ in a particular direction, is proportional to the radius of the momentum surface in that direction divided by the optical angular frequency. For each direction of propagation there are two possible eigen-phase-velocities, with corresponding orthogonal, eigen-polarizations as solutions to equation (17). The two surfaces intersect in degenerate directions known as the optical axes, and there may be up to four such intersections in biaxial media, which lie along two optical axes in the xz plane. For uniaxial materials there are only two such intersections along a single line, giving a single optical axis. For the case of TeO_2 optical activity breaks the degeneracy along the optical axis, and results in a slight splitting of the two eigen-velocities along the optical axis, as shown in Figure 2. In this case the eigen-polarizations are circular along the optical axis, and become elliptical as the direction of propagation moves away from the optical axis.

The impermeability tensor $\eta_{ij}=\varepsilon_0(\varepsilon^{-1})_{ij}$ is the inverse of the dielectric tensor ε and is given by the relation $\varepsilon^{-1}{}_{kj}\varepsilon_{ik}=\delta_{ij}$. As a second rank symmetric tensor it describes a quadratic surface known as the index ellipsoid

$$(\varepsilon^{-1})_{ij}\, x_i x_j = \left(\frac{1}{n^2}\right)_{ij} x_i x_j = 1 \tag{19}$$

In the principal coordinate system this equation reduces to the familiar representation of a general ellipsoid.

$$\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} = 1 \tag{20}$$

This surface is a convenient geometric representation for finding the optical eigenmodes of the displacement vector **D** for a given direction of propagation. These eigenmodes are found by finding the principal axes of the ellipse normal to the propagation direction. Associated with each eigenmode is a corresponding index of refraction, equal to the ellipse radius along each principal axis.

II.3 Photoelastic coupling

The photoelastic effect is usually described in terms of an elastically induced perturbation of the impermeability tensor mediated by a fourth rank elastooptic tensor, which has nonzero components in all materials [3,6].

$$\Delta\eta_{ij} = \Delta\left(\frac{1}{n^2}\right)_{ij} = p_{ijkl} s_{kl} \tag{21}$$

The symmetry of the index ellipsoid in i and j, and the strain tensor in k and l result in the symmetry relations for the strain-optic tensor $p_{ijkl}=p_{jikl}=p_{ijlk}=p_{jilk}$. Because the index perturbations due to the photoelastic effect are small we can use the relationship $\delta n=-(1/2)n^3 d(1/n^2)$ to write

$$\Delta n_{ij}=-\{n^3/2\}p_{ijkl}\, S_{kl} \tag{22}$$

Thus the perturbation of the optical index is proportional to the magnitude of the applied strain, and as long as the appropriate photoelastic tensor coefficient is nonzero there will be a resulting phase modulation of a properly polarized optical wave passing through the medium.

An acoustic eigenmode plane wave as in Eq. (1), will induce a strain wave of the form

$$\vec{S}(\vec{x},t) = \frac{1}{2}W\hat{S}_{mn}\cos(\Omega t - \vec{K}\cdot\vec{x}) \tag{23}$$

Where $\mathbf{S}_{mn}$ is a unit strain tensor for the given mode. This will induce a periodic traveling wave volume dielectric tensor perturbation that will couple the ith polarization component of the input mode with the j^{th} polarization component of the output mode

$$\Delta\varepsilon_{ij}(\vec{x},t) = \frac{W}{2}\varepsilon_{ik}(p_{klmn}\widehat{S_{mn}})\varepsilon_{lj}\cos(\Omega t-\vec{K}\cdot\vec{x}) \tag{24}$$

For an isotropic medium this corresponds to a perturbation of the index of refraction given by

$$\Delta n_{ij}(\vec{x},t) = -\frac{n^3}{2}Wp_{klmn}\widehat{S_{mn}}\cos(\Omega t-\vec{K}\cdot\vec{x}) = \delta n_0\cos(\Omega t-\vec{K}\cdot\vec{x}) \tag{25}$$

This index grating of amplitude $\delta n_0=-\{n^3/2\}Wp_{ijmn}\mathbf{S}_{mn}$ can diffract an incident optical plane wave, of polarization *i* into a diffracted beam with polarization *j*, if the appropriate photoelastic tensor element is nonzero, and as long as both energy and momentum are conserved. This results in two conservation equations for the incident and diffracted optical waves.

$$\vec{k}_d = \vec{k}_i \pm m\vec{K}_a \quad m = 0,1,2,\ldots N \text{ (momentum conservation)} \tag{26}$$
$$\omega_d = \omega_i \pm m\Omega_a \quad m = 0,1,2,\ldots N \text{ (energy conservation)} \tag{27}$$

The plus and minus signs in these equations correspond to the annihilation or creation of m phonons, respectively.

With near normal incidence, many orders are simultaneously created and we are said to be in the Raman-Nath regime, which is illustrated in Figure 3a. When the width of the acoustic wave in the direction of light propagation greatly exceeds the characteristic length $L_0=\frac{n\Lambda_0^2\cos(\theta)}{\lambda}$, indicating that an optical beam encounters several acoustic wavefronts as it traverses the acoustic wave at an angle θ, then the interaction is in the Bragg regime. When the incident light is at the Bragg angle with respect to the acoustic wave, then only one diffracted order is allowed with either $m=\pm 1$, which is illustrated in Figure 3b. This describes a quantum mechanical particle scattering interaction between three particles, the incident photon, the diffracted photon, and the acoustic phonon. In order to conserve energy the diffracted photon is Doppler shifted by the moving phase grating by an amount exactly equal to the frequency of the acoustic phonon. This frequency shift can only be observed interferometrically, and it plays a key role in the heterodyne detection experiment in the lab. The momentum matching equation (26) describes a closed triangle in momentum space whose vertices, corresponding to the incident and diffracted wave, must be allowed optical eigenmodes. This is illustrated in Figure 4a for the isotropic case, such as would be observed in fused silica, in which case the closed triangle is isosceles and $|\mathbf{k}_i|=|\mathbf{k}_d|$, because the change in the optical frequency and energy due to the acoustic wave is less than one part in 10^6. The angles between the acoustic plane wavefronts and the incident and diffracted optical beams are the same, and are called the Bragg angle θ_B. This direction corresponds to constructive interference between different portions of the diffracted beam. It can be derived by modelling each acoustic wavefront as a partially reflecting mirror, and requiring the reflected portion from adjacent wavefronts to add up in phase as is illustrated in Figure 4b. The resulting Bragg angle is given by

$$\theta_B=\sin^{-1}\left(\frac{|K|}{2|k|}\right)=\sin^{-1}\left(\frac{\lambda}{2n\Lambda}\right)=\sin^{-1}\left(\frac{\lambda f}{2v_a n}\right)\approx\frac{\lambda f}{2v_a n} \tag{28}$$

The angular deviation of the diffracted light is seen to be proportional to the frequency of the applied acoustic wave in the small angle approximation, which is the basis of many acousto-optic systems.

In an anisotropic medium the optical and acoustic momentum vectors can be a function of direction and polarization. The acousto-optic interaction can result in a change of the state of polarization of the diffracted optical wave as well as the direction of propagation. The technologically most relevant case is when the incident optical polarization is in the slow mode, and an acoustic mode with the proper photoelastic tensor coefficient is used to switch the state of polarization to the fast mode[7]. This case is illustrated in Figure 5a for a positive uniaxial crystal, where the angle of incidence θ_1 has a corresponding extraordinary index $n_s(\theta_1)=n_e(\theta_1)$, and the angle of diffraction θ_2 has a corresponding ordinary index $n_f(\theta_2)=n_o$. This type of birefringent diffraction can result in an increased bandwidth for a given efficiency by using tangentially degenerate phase matching, where the midband acoustic momentum vector is parallel with the nearest part of the inner optical momentum surface. For an acoustic beam rotated by an angle β, the angles of incidence $\theta_i=\theta_2 - \beta$, and diffraction $\theta_d=\theta_1 + \beta$, measured with respect to the normal to the acoustic beam, are the appropriate coordinates for describing the interaction. The momentum matching condition requires the acoustic momentum to have a magnitude which closes the triangle in momentum space.

$$|K_a| = \frac{2\pi}{\Lambda} = k_s(\theta_i)\sin(\theta_i)+k_f(\theta_d)\sin(\theta_d)= \frac{2\pi n_s(\theta_1)}{\lambda}\sin(\theta_i) + \frac{2\pi n_f(\theta_2)}{\lambda}\sin(\theta_d) \qquad (29)$$

The optical momentum wave vectors must also be conserved orthogonal to the rotated acoustic beam which leads to an expression for the full angle through which the light is diffracted.

$$\sin(\theta_i+\theta_d)= \sin(\theta_1+\theta_2) = \frac{\lambda_0}{n_s(\theta_1)\Lambda}\cos(\theta_d) = \frac{\lambda_0}{n_f(\theta_2)\Lambda}\cos(\theta_i) \qquad (30)$$

For an acoustic plane wave propagating along the optical symmetry axis with $\beta = 0$, the expressions for the required angle of incidence and diffraction of a polarization switching anisotropic diffraction mechanism are modified from the isotropic or polarization preserving relationship, yielding the Dixon equations[7].

$$\begin{aligned} \sin(\theta_i)&= \frac{\lambda}{2n_s\Lambda}\left(1+\frac{\Lambda^2}{\lambda^2}(n_s^2-n_f^2)\right) = \frac{\lambda f}{2n_s v_a}\left(1+\frac{v_a^2}{f^2\lambda^2}(n_s^2-n_f^2)\right) \\ \sin(\theta_d)&= \frac{\lambda}{2n_f\Lambda}\left(1+\frac{\Lambda^2}{\lambda^2}(n_f^2-n_s^2)\right) = \frac{\lambda f}{2n_f v_a}\left(1+\frac{v_a^2}{f^2\lambda^2}(n_f^2-n_s^2)\right) \end{aligned} \qquad (31)$$

These relations are plotted for the same birefringence indicated in the anisotropic Bragg matching diagram in Figure 5. The broad minimum of θ_i can be used as a fixed incidence angle which yields a wide range of diffraction angles for different input frequencies. When the slow and fast modes are degenerate, $n_s(\theta_1)=n_f(\theta_2)$, then this is seen to reduce to the familiar Bragg diffraction condition of isotropic media where $\theta_i=\theta_d$.

When the acoustics are also anisotropic, then v_a varies with the acoustic beam angle β, and the Bragg matching condition is most easily solved for graphically, as illustrated in Figure 6. In this figure the diffracted optical momentum vector is found at the intersection of the optical normal surface with the perturbing acoustic normal surface of the appropriate scale for the input frequency which is centered on the input optical momentum vector eigenmode. The acoustic momentum surface can be approximated with a Taylor series expansion in the vicinity of the crystal symmetry axis, and to second order the acoustic momentum vector can be represented as an asymmetric paraboloid.

$$\left|K_a(\beta_y,\beta_z)\right| =(1 + a_y\beta_y + a_z\beta_z - b_y\beta_y^2 - b_z\beta_z^2)\frac{2\pi f}{v_0} \qquad (32)$$

In this expression β_y is the off axis angle in the height direction, β_z is the off axis angle in the interaction plane, and v_0 is the acoustic velocity along the symmetry axis. The linear terms in β_y and β_z represent on axis acoustic walkoff effects, and are identically zero if the transducer face is accurately aligned normal to an acoustic symmetry axis. The intersection of this paraboloid with the optical normal surface gives the locus of exactly phase matched momentum vectors of the diffracted optical field, and this is illustrated in the random dot stereo pair shown in Figure 7a. In the x-z interaction plane we can derive a relationship between the incident and diffracted optical angles with the acoustic off axis angle β_z.

$$n_f\sin(\theta_1) - n_s\sin(\theta_2) = n_f(\theta_2)\sin(\theta_d - \beta_z) - n_s(\theta_1)\sin(\theta_i - \beta_z) = (1 - b_z\beta_z^2)\frac{\lambda f}{v_0} \qquad (33)$$

This relationship, coupled with Eqs. 29-30, allows us to uniquely solve for the diffraction angle in arbitrarily anisotropic media. However, from the systems point of view it usually suffices to assume a linear relationship between input frequency and diffraction angle. In tellurium dioxide the acoustic anisotropy of the anomalously slow shear [110] mode is so large that these effects must be considered in attempting to explain non ideal AOD operation, such as frequency plane scale distortion and frequency plane blur.[8] An illustration of the normal surface intersection for the case of the TeO_2 slow shear mode with a greatly exaggerated frequency is shown as a random dot stereo pair in Figure 7b. The Schaeffer-Bergman diffraction pattern is given by the momentum conserving interactions in a uniformly excited acoustic medium, which are just the intersection of the diffracted wave vector momentum surface with the perturbing acoustic momentum surface, giving the locus of phase matched diffraction allowed eigenmodes.

II.4 1-Dimensional Coupled Mode Solutions

One method of solving the 1-dimensional acousto-optic coupled mode equation is to assume a perfectly phase matched interaction, while also assuming that the modal amplitudes are only a function of the z coordinate, even though this is not completely rigorous. For an incident optical plane wave with polarization $\mathbf{E}_0$ and incident amplitude $A_0(0)$, that is diffracted by the acoustic wave into a plane wave with polarization $\mathbf{E}_1$ whose amplitude grows with interaction distance z within the AO medium, the total field can be written in the interaction regime 0< z <L, as

$$\vec{E}(\vec{x},t) = A_0(z)\,\hat{E}_0 e^{-i(\omega_0 t - k_{x0}x - k_{z0}z)} + A_1(z)\,\hat{E}_1 e^{-i(\omega_1 t - k_{x1}x - k_{z1}z)} \qquad (34)$$

In the presence of the dielectric perturbation given by Eq. (24) the two modes are coupled, provided that momentum and energy are conserved. The total field is a solution of the inhomogeneous wave equation

$$\nabla^2\vec{E}(\vec{x},t) - \mu(\varepsilon + \delta\varepsilon(\vec{x},t))\frac{\kappa^2\vec{E}(\vec{x},t)}{\kappa t^2} = 0 \qquad (35)$$

The individual fields with constant amplitudes are solutions of the unperturbed homogeneous wave equation which allows us to substitute the total field and the acoustically induced dielectric perturbation in the wave equation, and obtain for the one dimensional interaction configuration

$$\leftarrow_{m=0,1} \hat{E}_m e^{-i(\omega_m t - k_{xm}x - k_{zm}z)}\left[\frac{\kappa^2}{\kappa z^2} - 2ik_{zm}\frac{\kappa}{\kappa z}\right]A_m(z)$$
$$+ \frac{W}{2}\varepsilon_{ik}(p_{klmn}\hat{S}_{mn})\varepsilon_{lj}\left[e^{-i(\Omega t - K_x x - K_z z)} + c.c\right]m \leftarrow_{l=0,1} \omega_l^2 A_l(z)\hat{E}_l e^{-i(\omega_l t - k_{xl}x - k_{zl}z)} = 0 \qquad (36)$$

The adiabatic condition allows us to assume that the modal amplitudes $A_m(z)$ are slowly changing in z compared to an optical wavelength so that we can neglect the second order spatial derivative. We now require that the coefficients of $\hat{E}_m e^{-i(\omega_m t - k_{xm}x - k_{zm}z)}$ vanish for m=0,1, in order to obtain a phase synchronous transfer of power between the modes. This requires that $k_{x1}x - k_{z1}z = k_{x0}x - k_{z0}z$ □ $K_x x - K_z z$ and $\omega_1 = \omega_0$□Ω, which are the familiar momentum and energy conservation conditions.

For Bragg interactions only a single order has a totally phase synchronous interaction with the input fields so we only get coupling to a single mode. This results in the well known coupled mode equations for perfectly phase matched interactions.

$$dA_0(z)/dz = -i\kappa \; A_1 \qquad (37)$$
$$dA_1(z)/dz = -i\kappa^* A_0$$

Where the coupling constant is found from the incident and diffracted polarization vectors left and right projected onto the dielectric perturbation tensor.

$$\kappa = \frac{\omega}{4}\hat{E}_0^* \cdot \Delta\varepsilon \hat{E}_1 = \frac{\omega}{4}\left| E_{0i}^* \cdot \frac{W}{2} \varepsilon_{ik}(p_{klmn}\hat{S}_{mn})\varepsilon_{lj} E_{1j} \right| \qquad (38)$$

The solution of the coupled mode equation with the implied boundary condition of $A_1(0)=0$, yields the perfectly coupled mode solution for z>0

$$A_0(z) = A_0(0)\cos(|\kappa|z) \qquad (39)$$
$$A_1(z) = -i\{\kappa/|\kappa|\}\, A_0(0)\,\sin(|\kappa|z)$$

So the optical field is seen to oscillate back and forth between the incident and diffracted field each distance $2L_m=\pi/|\kappa|$, as the beams propagate in the z direction. Complete power transfer occurs from the input to the diffracted mode after a distance of z propagation given by L_m. The diffraction efficiency is the ratio of the incident optical intensity to the intensity transferred to the diffracted beam in a distance L, and it is given by

$$\frac{I_2}{I_1} = \frac{|A_1(L)|^2}{|A_0(0)|^2} = \sin^2(|\kappa L) \qquad (40)$$

Thus given the amplitude W of the acoustic mode $\hat{S}_{kl}$, the interaction length L, the input polarization $\hat{E}_0$, the output polarization $\hat{E}_1$, and the material tensors ε_{ij} and p_{ijkl}, we can find the percentage of light diffracted into the output beam.

II.5 Acousto-Optic Devices

An acousto-optic device is constructed by bonding an acousto-electric transducer onto a photoelastic medium, so that acoustic waves can be launched into the medium, and is illustrated in Figure 8. The transducer is usually a piezoelectric crystal of thickness t_0, metalized on both faces so that an electric field can be applied transversely in the k direction. This induces a strain through the third rank piezoelectric tensor **d**, which can only exist in non-centrosymmetric materials.

$$S_{ij}(t) = d_{ijk}E_k = d_{ijk}v_k(t)/t_0 \qquad (41)$$

The appropriate choice of transducer crystal cut and orientation are used to produce the desired polarization of the acoustic wave which is launched into the photoelastic medium. The time dependent strain within the transducer is coupled into the photoelastic medium with a frequency dependent efficiency dictated by the acoustic impedance matching of the transducer and bonding layers. The frequency dependence of the electro-acoustic conversion R(f), is due to the electrical matching network, the transducers resonant bandwidth and the mechanical coupling efficiency bandshape. By applying a sinusoidally varying electric field to the transducer, that is within its acoustic resonant bandwidth, we can launch a propagating acoustic wave into the photoelastic medium. Since the transducer has a finite spatial aperture the harmonic acoustic field will have a spatial angular divergence. The angular spectrum of the transducer is given by the Fourier transform of its aperture p(y,z), scaled by the appropriate acoustic wavelength. The angular divergence of the transducer in the interaction dimension coupled with the phase matching condition is what determines the acousto-optic bandshape of the device. For an isotropic device with a simple uniform rectangular transducer of length L and height H the transducer radiation pattern acoustic angular spectrum in the interaction dimension is given by the simple 1-D transform $\mathrm{sinc}^2(L\beta_z/\Lambda)$.

The resulting isotropic acousto-optic bandshape can be derived from the viewpoint of phase mismatched interaction, or from the viewpoint of selecting the appropriate angular component from the transducer radiation field in order to obtain perfect phase matching, and these different interpretations are schematically illustrated in Figure 9a. The isotropic acousto-optic bandshape is given in terms of the perfectly matched on axis frequency f_m, normalized by the center frequency f_0, and the normalized interaction length, as a function of the normalized frequency $F=f/f_0$.

$$W(F) = \mathrm{sinc}\left[\frac{L}{2L_0} F(F_m - F)\right] \qquad (42)$$

Thus the transducer length L determines the acousto-optic bandwidth, and it determines the effective interaction length as well, thereby affecting the diffraction efficiency.

In the tangentially degenerate approach to birefringent phase matching[9], the acoustic wave vector is tangent to the locus of fast diffracted wave vectors at the symmetric center frequency f_1, and the peak of the transducer angular spectrum intersects the diffracted wave vector surface at two frequencies $f_m^{\pm} = f_0 \pm \Delta f/2$. The resulting birefringent acousto-optic bandshape is broadened, and is given in terms of normalized frequency variables.

$$W(F) = \mathrm{sinc}\left[\frac{L}{2L_0} (F - F_1)^2 - \frac{\Delta F}{2}\right] \qquad (42)$$

When the symmetrical frequency is at midband and phase matched $\Delta f=0$, then $f_m=f_0=f_1$, and the tangentially matched acousto-optic bandshape has a simple quadratic frequency dependent phase mismatch term $(F-1)^2$. The decreased transducer angular bandwidth needed by a device operating in the tangentially degenerate phase matching regime is illustrated in Figure 9b. The transducer can be longer and a larger interaction length results in a greater diffraction efficiency, and the bandshape becomes symmetrical.

The transducer height H determines the degree of collimation in the orthogonal dimension which determines the diffraction limited usable aperture time. For the isotropic acoustic case the transducer radiation pattern remains within the near field in the height dimension for a distance $D=H^2/\Lambda$, which is equal to the distance that the transducer angular spectrum at the 4dB half width intersects the transducer geometric shadow. In the anisotropic acoustic case the near field in the height dimension is modified by the acoustic curvature factor giving a near field distance $D'=H^2/\Lambda\ (0.5 - b_y)$. The transducer area LH relates to the acoustic power density through the equation $P_a=(1/2)\rho_m v_a^2|S|^2LH$. and thus determines the optical diffraction efficiency in terms of the electrical power input to the transducer. A tradeoff between the diffraction efficiency and the interaction bandwidth can be tailored by an appropriate choice of the transducer length to height ratio L/H.

A Bragg cell or acousto-optic deflector (AOD) is the basic electrical to optical transducer used in this lab. From the system point of view this device is modelled as a 1-D traveling wave modulator with a finite aperture window. Thus when a temporal signal s(t) is applied to the device, the resulting transmission function of an idealized device is represented as

$$t(x,y)=\alpha\ s(t-x/v_a)w(x) \tag{43}$$

The acoustic velocity v_a is taken to be the nominal velocity along the symmetry axis normal to the transducer. The window function w(x) is the product of the device finite aperture window with the acoustic attenuation, and the input optical beam Gaussian profile is often included as well, thereby making the window function a hybrid window of the induced polarization field which is the product of input field and dielectric perturbation. This device model is sufficient for simple system calculations, but it ignores the nonideal behavior of a real device as discussed in this section. In considering the proper use of a device in a system, it is necessary to minimize the effects of its nonideal behavior, such as its polarization response,[10] off axis diffraction effects,[8] and intermodulation nonlinearities.[11] A more complete description of a Bragg cell response[12] is based on a superposition of its response at each frequency across the device bandwidth. When we apply a broadband signal s(t) to the transducer each of its Fourier components launches a spatially diffracting harmonic traveling wave into the photoelastic medium. The temporal Fourier decomposition of a single sideband of the input signal is given by

$$\tilde{s}(t) = \int \tilde{S}(f)e^{i2\pi ft}df \tag{44}$$

The device response to an individual Fourier component is a frequency dependent acoustic diffraction integrated along the direction of optical propagation. Thus we can write a more general description of the diffracted field from the device for an input plane wave at the Bragg angle.

$$a_d(x,y,z,t) = e^{-i2\pi[\nu t+k_z z]}i\int t_f(x,y)\ \tilde{S}(f)e^{i2\pi ft}dt \tag{45}$$

The function $t_f(x,y)$ is the frequency dependent amplitude transmission of the Bragg cell. It can be obtained from the 3-D acoustic Sommerfeld diffraction integral, integrated along the direction of optical propagation to form a 2-D projection of the acoustic pressure density field.[13,14,15]

The momentum space representation of the amplitude transmission of the device is given by the spatial Fourier transform of $t_f(x,y)$, and it is exactly what would be observed at the Fourier plane of an AO spectrum analyzer illuminated by a plane wave[16], and it more clearly illustrates the dependence of the device behavior on the physical mechanisms discussed previously in this section.

$$T_f(k_x,k_y) = \int t_f(x,y)\, e^{-i2\pi[k_x x+k_y y]}\, dx\, dy = \eta R(f) W(f) S(k_x, k_y) \qquad (46)$$

The constant $\eta=\Delta n\pi\, L/\lambda\cos\theta_0$ is just the amplitude diffraction efficiency derived from the coupled mode model and the transducer geometry, with a skewed interaction length $L/\cos\theta_0$. The acousto-electric conversion efficiency R(f) is the frequency and phase response of the tuning network, the transducer and the coupling to the photoelastic medium. The acousto-optic bandshape W(f) is determined by the type of diffraction ,either normal or birefringent, and upon the direction of propagation of the input wave and the transducer orientation and shape. The spatial frequency angular spectrum of the device, $S(k_x,k_y)$, is the result of the phase matching conditions applied to the traveling wave anisotropic acoustic propagation.

For the ideal traveling wave modulator device model of Eq. (42), the frequency dependent spatial frequency spectrum takes an especially simple form.

$$T_f(x,y) = \eta e^{i\pi A[k_x - f/v_a]}\, \mathrm{sinc}\left[A\left(k_x - \frac{f}{v_a}\right)\right] \delta(k_y) \qquad (47)$$

This equation represents a single diffracted spot in the Fourier plane, whose frequency resolution is limited by the device aperture in the acoustic propagation dimension. The phase factor represents the plane wave acoustic phase delay accumulated in the propagation to the center of the device where the optical axis is located. The term $\delta\,(k_y)$ represents the Fourier transform of an infinitely high transducer, and in general it will be replaced by a 2-dimensional Fourier transform of the transducer aperture, $P(\beta_y/\Lambda\ ,\ \beta_z/\Lambda\})$. For an input optical field propagating in the x-z plane the transducer angular spectrum is sampled along the $k_z=0$ slice, equivalently projecting the 3-D diffracting acoustic wavefront by an integration of the acousto-optic modulation along the path of optical propagation, via the projection slice theorem.

For an anisotropic medium, the acoustic propagation has additional curvature factors which enter into the phase matching conditions, and the acoustic propagation equations yielding a more complex spatial frequency angular spectrum.

$$S_{k_z}(k_x, k_y) = \int_0^A e^{ix\frac{\Lambda_0}{2}[K_y^2(b_y-\frac{1}{2})+K_z^2(b_z-\frac{1}{2})]} P(K_y,K_z)dx = e^{i\pi A\,[k_x - f/v_a + \frac{\Lambda_0}{2}[K_y^2(b_y-\frac{1}{2})+K_z^2(b_z-\frac{1}{2})]]}$$

$$\mathrm{sinc}[A\,[k_x - f/v_a + \frac{\Lambda_0}{2}[K_y^2(b_y-\frac{1}{2})+K_z^2(b_z-\frac{1}{2})]] \qquad (48)$$

The curvature factors $(b_y-1/2)$ and $(b_z-1/2)$ are the anisotropic curvature factors of the energy flow or acoustic Poynting vectors, and are derived as the normal vector of the momentum surface. This represents a curved locus in the Fourier domain weighted by the finite aperture sinc in the propagation dimension and the transducer angular spectrum in the height dimension. When a collimated plane wave is incident on the device at a specific **k** then a slice out of this spatial frequency spectrum is sampled by the integrating optical propagation.

In some experiments a more complicated mode of operation is utilized, where the optics is focused into the center of the acoustic column. In this case the individual slices of the spatial frequency spectrum will generate diffraction components due to each plane wave component of the input beam which must be summed up to find the full diffracted wavefront. An additional complication in anisotropic media is the fact that the acousto-optic bandshape function is also a function of the input optical propagation direction $W_{\mathbf{k}}(f)$, and as the off axis angle increases the frequency corresponding to the tangential degeneracy also increases. In optically active media such as TeO_2[17,18] the polarization states of the diffracted optical wave also depend on the off axis propagation angle because the birefringence begins to dominate the degeneracy splitting of the optical rotary power. These effects are illustrated for off axis propagation angles of 5 and 12 degrees in TeO_2 in Figure 10., with the appropriate birefringence and rotary powers interpolated for 835 nm wavelength light[19], and these figures should be compared with the on axis optical normal surface plotted in Figure 2. A final complication is the dependence of the effective photoelastic constant and the coupling efficiency on the angle of propagation through the interaction medium. To minimize these off axis diffraction effects a limited range of off axis angles should be used to illuminate a slow shear TeO_2 Bragg cell.

Another technique used to increase the efficiency-bandwidth product of a deflector is to use a beamsteering acoustic phased-array-transducer. These transducers produce a narrow angular radiation lobe, ideal for high diffraction efficiency, but generally poor for bandwidth, and then they steer this acoustic radiation lobe as the frequency is varied in the direction necessary to phase match over a wide bandwidth. Two types of phased arrays are commonly used, the constant phase delay phased arrays, and the constant time delay phased arrays, and these are illustrated in Figure 11. A constant phase delay of 180 degrees between adjacent transducer array elements can be accomplished by applying the RF to alternate transducers which lie on top of a voltage dividing floating ground plane. Such an array produces two main acoustic lobes which both steer towards the normal as the frequency is increased. Only one of these lobes is utilized in a typical beam steering acoustooptic device, and the power in the other lobe is wasted. When 3 RF phases 120 degrees apart are used to drive 3 alternating transducer phases, then only one main acoustic beam is produced, thereby doubling the efficiency of such a device, however the complexity of 120 degree hybrids detracts from the attractiveness of this scheme. An alternate approach to producing a single acoustic beam which steers with frequency is the constant time delay phased array implemented as a stepped staircase transducer, where the step size is half the midband acoustic wavelength $\Lambda_c/2$ as illustrated in Figure 11b. The adjacent transducers are also driven 180 degrees out of phase in this case. In both of these beam steering techniques the locus of acoustic momentum vectors trace out a straight line orthogonal to the transducer, but offset by the fundamental grating frequency of the transducer, so that the resulting acoustic momentum vector is given by $\mathbf{K}_a = 2\pi v_a/f\hat{\mathbf{z}} + \pi/d\hat{\mathbf{x}}$, where f is the applied frequency, d is the width of each phased array element, and $\hat{\mathbf{x}}$ is in the direction of the transducer elements. As the Bragg cell is rotated this locus of acoustic beams is brought into tangency with the optical momentum surface, in an analogous manner as in anisotropic diffraction, thereby maintaining a small phase mismatch over a wide bandwidth because of the quadratic deviation from perfect phase matching. However, when such a beam steering array is rotated to the other Bragg angle, then the locus of acoustic momentum vectors cuts through the optical momentum surface with a linear dependence of the momentum mismatch on frequency, seriously degrading the bandwidth for that order.

II.6 Acousto-optic Modulators

Another important device is the acousto-optic point modulator or AOM, which is similarly constructed by bonding an acousto-electric transducer to a photo-elastic medium. The principal difference between an AOM and an

AOD is that the AOM is used with a tightly focused optical spot incident on the center of the acoustic column and the AOD is illuminated with a collimated wave. This is because the modulator must have a quick access time in order to achieve wide modulation bandwidth, which is the inverse of the acoustic propagation time across the beam waist of the focused spot[20]. In order to be able to modulate each spatial frequency component of the focused optical field there must be a corresponding phase matched frequency component of the diffracting acoustic wavefront, thereby dictating a short transducer length L. This leads to a tradeoff between diffraction efficiency and access time. The Bragg angle centered focusing incident beam and the diverging diffracted wave must be separated in the Fourier plane for single sideband suppressed carrier modulation, where the diffracted intensity is proportional to the envelope modulation of the acoustic carrier. This leads to a requirement that the carrier frequency be several times higher than the access time limited modulation bandwidth, which in turn requires that several acoustic phase fronts be contained within the optical beam waist. The intensity modulation of the diffracted beam by the envelope of the acoustic signal can be considered to be a mixing of the carrier with the sidebands which are responsible for the amplitude modulation. When the modulation frequency increases the diffraction angle of the sidebands increases so that less of the diffracted cones from the sidebands overlap with the carrier. Optical heterodyning is optimized when the interfering components are colinearly propagating, so as the modulation frequency increases the modulation depth decreases, which is an alternative derivation of the modulator bandwidth limitation. This type of modulator arrangement is illustrated in Figure 12, and the overlap of the diffracted waves in momentum space is illustrated for a frequency near the modulator 3 dB bandwidth limitation.

An alternative modulation geometry that results in a phase modulated optical wave is obtained when the optical beam waist is much smaller than an acoustic wavelength, and is illustrated in Figure 13. In this case the optical beam sees at any one time an instantaneous acoustic density which homogeneously slows down the optical wave uniformly across the aperture of the focused beam. As an example of this regime of operation, consider $LiNbO_3$ which has a longitudinal acoustic velocity of 6.57 mm/μsec. This indicates that at an acoustic frequency of 50 MHz the acoustic wavelength will be 131.4 μm, and the optical wave can easily be focused to a much smaller 10-30 μm spot. This type of phase modulation requires the production of a large number of Bessel function weighted temporal sidebands, which are a characteristic of Raman-Nath regime AO diffraction, and since the optical wave passes through less than one acoustic wavefront it is no longer a Bragg regime modulator. This type of acoustic undulation modulator is similar to an electrooptic modulator, and the phase modulated signal can be considered to be produced by the interference of the appropriately weighted temporal sidebands, which are essentially colinear spatially. A short transducer is again required in order to contain within its angular bandwidth appropriate components needed to diffract different optical momentum components several times in order to produce the appropriately weighted sidebands. An acoustic phase modulator of this nature is only useful in the context of an interferometric system, because the intensity is unmodulated by the passage of an acoustic wave, and the diffraction angle is far less than the optical momentum width.

A Bragg cell can be used as a modulator as well as a deflector, but because of the long transducers that are utilized with narrow acoustic angular spectrums a limit is placed on the amount of focusing of the incident optical wave that is allowed in order to have a phase matched acoustic component for each momentum component of the input optical field. This places a limit on how small of an aperture that can be illuminated and therefore limits the modulation bandwidth, or inverse access time. In order to use an AOD as a phase modulator the incident light should be collimated and incident at the Bragg angle. When pulsed laser illumination is used, access time is not a constraint, and the Bragg cell can be loaded with a phase shifted sinusoidal tone on each pulse. The diffracted phase modulated and doppler shifted plane wave is easily separated from the undiffracted wave in the Fourier plane since the AOD is deep within the Bragg regime and illuminated with a plane wave. This type of modulation arrangement is illustrated in Figure 13.

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IIIa. Preparation

Read some of the acoustooptic references in the lab I. C. Chang, Acousto-optic interactions - a review: I. Acousto-optic devices and applications, IEEE trans. Sonics and Ultrasonics, vol. SU-23(1), p. 2 (1976).

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IIIb. Prelab

1. Calculate the internal and external Bragg diffraction angles for an acoustic wave propagating normal to one of the faces illuminated through an orthogonal face by a coherent laser beam. Your equation for the external angle should be independent of the index of refraction of the crystal. Why is this? (Hint: What quantity is conserved across a dielectric interface?) Will this be true in general?

Calculate the external Bragg angle for the following two cases in the case of illumination by a HeNe laser beam of wavelength .6328 μm:

> a) A Lead Molibdinate crystal with an acoustic velocity 3.6 mm/μsec driven at an RF frequency of 140 Mhz.
>
> b) A flint glass Bragg cell with an acoustic velocity of 3.96Imm/μsec driven at an RF frequency of 85 Mhz.

2. *K-Space:* Sketch the index ellipsoid of a uniaxial birefringent crystal (for the purposes of this exercise, you may wish to exaggerate the amount the birefringence.) Since the momentum of a photon is equal to the propagation constant k which is in turn proportional to the index of refraction, the index ellipsoid is proportional to the locus of allowed photon

momenta within the crystal. This "k-space" diagram is a useful tool for visualizing Bragg interactions.

The requirement of Bragg matched interactions is that the vector momentum be conserved. Thus, the input and output light momenta must lie on the surfaces you have drawn, and the acoustic momentum, K, must connect them. Use this graphical tool to analyze the following cases:

a) The input light is directed perpendicular to the optic axis and is polarized to lie on the inner momentum surface (smaller index). The acoustics are driven parallel to the optic axis. This type of interaction is referred to as "polarization switching." Why? There is an intrinsic 3 dB loss in this geometry. Why?

b) Consider now switching the input and output optical beams. That is, the input light now enters at an angle and on the outer index surface while the output light is directed perpendicular to the optic axis on the inner index surface. The acoustics remain directed parallel to the optic axis. Will this geometry suffer from the same 3 dB loss? This geometry is called "tangentially degenerate matching." Will it have a larger or smaller bandwidth (range of acoustic frequencies) than case a? Which of these would make a better acoustooptic deflector? Can you think of a way to get tangentially degenerate matching in an isotropic material such as glass?

3. A flint glass Bragg cell is used in an acoustooptic spectrum analyzer system, which is illuminated with collimated Bragg matched laser light from a HeNe laser with wavelength λ=632.8 nm, and the diffracted light is Fourier transformed with a lens of focal length. The acoustooptic device aperture is X=4 cm, the acoustic propagation velocity is v_a=3.96mm/μsec, the bandwidth of the transducer is B=40 MHz, from 50 Mhz to 90 MHz. What is the fundamental limit on the frequency resolution of this system, and how many such independently resolved frequency bins are achievable. What should be the detector spacing, and how many detector elements should be utilized, giving what size of detector array.

For the spectrum analyzer you have designed, estimate the number of equivalent digital floating-point operations per second (flops). Hint: the fast Fourier transform algorithm takes order N log(N) floating operations where N is the number of points being transformed. How fast is your analyzer in comparison to modern digital computers?

(Extra Credit)

4. An acoustic beam steering transducer uses 6 staircase stepped elements each with a width 8.5 mm and step height of $\Delta=\Lambda_c/2 = v_a/2f_c = 28.28\ \mu m$. The center frequency is f_c=70MHz and the transducer bandwidth is 40 Mhz. Each transducer step is driven 180 degrees out of phase with respect to its neighbors.

Write an equation for the complex acoustic amplitude at a plane after the transducer as a function of coordinate and applied RF frequency. What does this reduce to when $f = f_c$? Now find the corresponding angular frequency distribution by taking the Fourier transform of this distribution. It may help to think of this as a 2 dimensional Fourier transform. You should find that as the RF frequency is changed, the acoustic radiation pattern changes direction. This is called acoustic beam steering and is analogous to the techniques used in phased-array radars. Sketch on a k-space diagram the path of the maximum of acoustic radiation as the RF frequency is swept over the bandwidth. Why is beam-steering a good thing to do?

5. Imagine that light is incident normally on a Bragg cell (that is, the light and the acoustics are exactly perpendicular.) Derive an expression for the diffraction that will be seen when the Bragg cell is driven with a single RF frequency. How does this diffraction differ from the normal operation of a Bragg cell?

IV. Procedure

1. RF section.

Apply +24V power to the NEC RF modulator, and apply a positive bias to the AGC to adjust the output power. Hook up the HP 8111 pulse generator to the video input set up to produce a square wave and observe the RF output on an oscilloscope as you vary the AGC bias. What is the frequency of oscillation produced by the internal oscillator, and what is the effect of the input pulses? Set the offset and amplitude on the pulse generator to obtain a clean pulsed RF waveform, with as little modulation in the off time as possible. Determine the voltage at which the NEC modulator "switches". What is the rise time of the pulse generator, and the modulated RF output? What is the output power produced by the RF modulator as a function of the AGC bias voltage? Make sure it is less than 1 Watt peak in order to be sure of not blowing out the piezoelectric transducer. Apply this pulsed RF signal to the NEC acoustooptic modulator that is on a rotation stage.

2. Bragg alignment.

Spatial filter and collimate the HeNe laser, aperture down to a 1-2 mm pencil beam and align the beam onto the AO modulator. Find the Bragg diffraction by rotating, tilting and translating the AOM. Optimize the diffraction efficiency visually, by looking at a card placed 10-30cm beyond the AOM as you vary the alignment and identify the diffracted spot or spots. Often a good starting place for this alignment procedure is to come in at normal incidence with the HeNe, so that numerous diffraction orders are generated, both plus and minus, then to rotate to maximize the +1 order and suppress all of the others. Sketch your setup. How can you tell this is the +1 order? Measure the angle of diffraction and compare with your prelab calculation. Rotate to produce the -1 order and measure the diffraction angle again.

3. Acoustoptic modulation.

Block the undiffracted beam and place the power meter in the diffracted beam. Optimize the diffraction by maximizing the power in the diffracted beam. Measure the diffraction efficiency as a function of applied RF power which is controlled by varying the AGC bias. The diffraction efficiency is best measured by dividing the power in the diffracted beam by the power in the undiffracted beam *with no acoustics applied*. Plot your results, remember to correct for the duty factor of the RF pulses. Now place a high speed photodetector in the diffracted beam, and observe the detected output on an oscilloscope. It will probably be necessary to focus the light onto the detector surface in order to capture the entire diffracted beam.

What is the rise time that you were able to obtain with this configuration? How does this compare to the transit time of the acoustics across the beam? This is **not** the proper way to operate an acoustooptic modulator for high speed modulation, but it will work for low speed applications.

4. Access time-diffraction efficiency tradeoff.

Open up the aperture of the laser beam and place a good high F no. lens beyond the aperture and one focal length before the modulator so you are focusing to a tiny spot in the center of the AOM. Now observe the output plane as you open and close the aperture. Sketch and describe your observations. Is the diffracted output spot well separated from the undiffracted spot? Is the diffracted spot always circular, or does it change its shape for very wide apertures? The optimal setting for the aperture, in the sense of efficiency-rise time tradeoff, is the largest aperture at which the full beam is diffracted, and the rise-time is smallest. By using a high speed photodetector in the diffracted and undiffracted beam, measure the rise time and peak diffraction efficiency as a function of aperture size. Note the RF power level and AGC bias voltage. Do not confuse total power with diffraction efficiency, since you are varying the total power as you open and close the aperture. Now plot your results as a function of the spot size in the crystal and compare to the transit time. What is the optimal spot size and aperture setting in the sense of maintaining high diffraction efficiency and minimum rise time?

5. Acoustooptic deflector, or Bragg cell.

Look into the acoustooptic deflector and try to see the stair-stepped transducer. This tells you which way the acoustics are propagating. Replace your acoustooptic modulator with the acoustooptic deflector and illuminate with a wide aperture plane wave. Apply an RF signal from the HP 8601A to the AOD, which is in the range from 50-110 MHz, and with an amplitude of less than 1 Watt. Use a 2 inch lens to Fourier transform the output of the AO deflector, and observe the Fourier plane as you in-plane (Bragg) rotate the device. When the acoustooptic device is normal to the incident laser beam you should see orders on both sides of the undiffracted beam corresponding to the positive and negative diffracted orders. This is the Raman-Nath regime. When you rotate the device away from this position you should be able to optimize the diffraction efficiency into either of the diffracted orders. However, because this Bragg cell uses a stepped staircase acoustic beamsteering transducer, the bandwidths of the two orders will be entirely different. Set up the signal generator to produce a fast broad band sweep. Use the sweep ramp output as the x input to the oscilloscope, and place a wide area photodetector in the diffracted beam, and use the photodetector output as the y input to the oscilloscope. With the oscilloscope placed in x-y mode the pattern displayed is the acoustooptic band shape function. Does the plus or the minus order have larger band-width and does this agree with your expectation? Explain the "dip" in the center of the wider bandwidth - k-space diagrams may be helpful. Measure the band shape of the plus and minus orders and photograph, and notice that the band shape for either of the orders varies as you rotate the AO cell. Rotate the cell to the midband of the order that gives the widest bandwidth and largest diffraction efficiency.

6. Acoustooptic Spectrum Analyzer.
Place the line scan CCD array in the Fourier plane, where it will intercept the diffracted order from the AOD. Apply the required power to the CCD and observe the CCD output on the oscilloscope. Align the array for best focus and maximum detected power across the full AOD bandwidth. Observe the CCD output as you slowly scan the input frequency across the AOD bandwidth. Calibrate the detected location with the applied frequency. What is the frequency increment per pixel as the spot moves across the array, and what is the obtainable resolution of this acoustooptic spectrum analyzer system. Apply a pulsed RF waveform to the AOD, which is produced by mixing a pulse train produced by the HP8111 with an RF carrier produced by the HP8601A using an RF mixer, and observe the detected spectra. An RF mixer has three input ports, one for the local oscillator, one for the mixing waveform and one for the IF output, and it should be driven with signals of an appropriate amplitude, typically 10mW or less, have your TA show you how. From this spectra and your calibration measurements, calculate the modulator center frequency and pulse width, and compare with your other measurements of these parameters. Vary the pulse width and pulse repetition frequency and observe the effects on the detected spectra, and change to a sinusoidal waveform and observe the result on the CCD.

7. Heterodyne detection. (Extra Credit)
Use a beam splitter inserted before the AOD to split off a plane wave reference beam, and recombine this colinearly with the diffracted beam at the Fourier plane using another beam splitter in a Mach-Zehnder interferometer configuration. Place a high speed photodetector in this interference plane as you apply a single frequency from the HP8601A to the AOD, and observe the detector output. You should be able to see an interferometric reconstruction of the signal applied to the AOD, as long as the diffracted spot is hitting the detector, the reference beam is collinear, and the detector is fast enough. Block the reference beam and describe the change in the output. As you change the frequency from the signal

generator what happens to the output. Place a small opaque object such as a resistor wire in front of part of the detector as a bandpass filter, and describe the detected waveform as you sweep the diffracted light across the detectors face.

8. Time integrating correlator. (Extra Credit)

Place the AOM in the unexpanded laser beam before the Bragg cell, and align the system so that the diffracted beam produced by the AOM is rediffracted by the AOD. Image the diffracted beam from the AOD onto the time integrating CCD detector array, and block all undiffracted beams. Set up the HP8111 pulse generator to produce a pulse train of 200 ns pulses at a rate of 1 MHz, and use this as the external AM modulation to the HP8601A sweep generator, which is setup to produce a single tone. Apply this signal to both the AOM and the AOD and observe the time integrated correlation on the CCD output. Vary the PRF and duty factor of the pulse train and describe your results.