2.5 REFERENCE-BEAM INTERFEROMETERS:

These are interferometers where the object beam containing the signal is typically mixed with a planar reference beam derived directly from the same laser source. In this class are included (i) the homodyne Michelson, (ii) the heterodyne Michelson, interferometers and variants thereof.

2.5.1 Homodyne Michelson interferometer:

Consider the Michelson interferometer shown in fig. 2.2.

The two beams that are made to interfere are called the reference and object beams respectively, and can be expressed as:

$$E_R = a_R e^{-j(k_{opt} L_R - \omega_{opt} t)}$$
$$E_o = a_o e^{-j(k_{opt}(L_o + 2\delta(t)) - \omega_{opt} t)}$$

and the resulting interference intensity at the photodetector is:

$$P_D = P_R + P_O + 2\sqrt{P_R P_O}\cos\left\{k_{opt}\left(L_R - L_O\right) - 2k_{opt}\delta(t)\right\}$$

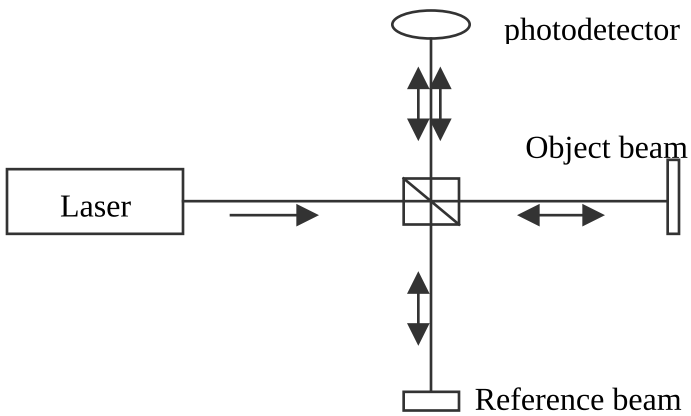

Figure 2.2: Michelson Interferometer

Here, we have intentionally re-written the phase term for the object beam as a ‘static’ part $k_{opt}L_O$ due to the static path length, and a time-varying part due to a time-varying normal displacement $\delta(t)$. In reality, even the static part is not quite static because of low frequency ambient vibration that can move the various components or the object around. If our signal of interest is high-frequency (say several kHz or higher) -- and this is the case for ultrasonic signals -- it is possible to use a stabilization system using a moving mirror on the reference ‘leg’ such that the static (or more appropriately, low frequency) phase difference is always actively kept constant. In fact, if the phase change due to our signal of interest $2k_{opt}\delta(t)$ is rather small –- as is the case for typical ultrasonic displacements –it may be wise to keep the static phase difference at quadrature, ie at $k_{opt}\left(L_R - L_O\right) = \frac{\pi}{2}$. As seen from Fig. 2.3, the interferometer displacement senstivity (for small displacements) is best at quadrature.

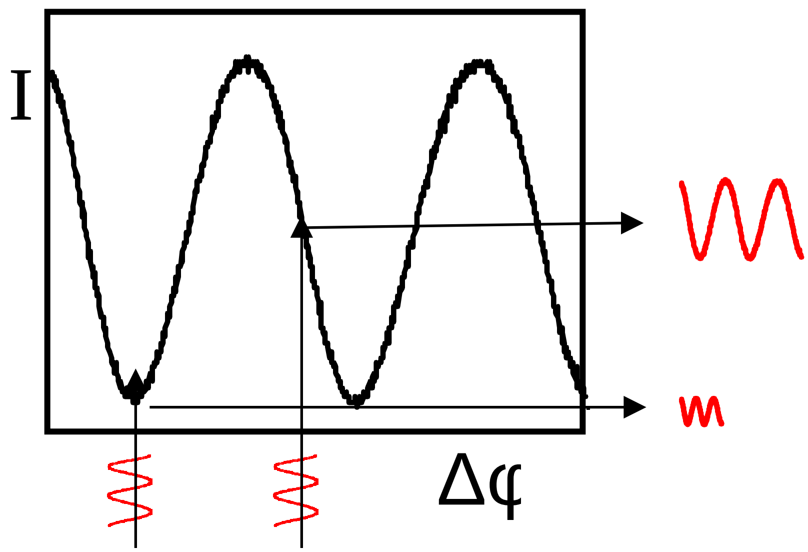

Figure 2.3: Michelson interferometer output intensity as a function of phase change.

The two beam Michelson interferometer that is stabilized at quadrature, therefore has an output intensity:

$$P_D = P_{tot}\left\{1 + M(2k_{opt}\delta(t))\right\}, \quad k_{opt}\delta <<1 \qquad \text{(QMI)}$$

where $P_{tot} = P_R + P_O$ is the dc-term

$M = \frac{2\sqrt{P_R P_O}}{P_R + P_O}$ is called the modulation depth of interference.

Let us assume that the object displacement is sinusoidal: $\delta(t) = \delta \sin\omega_u t$ where δ is the magnitude and ω_u represents the signal angular frequency ('u' for ultrasound). The shot-noise limited SNR of a stabilized Michelson interferometer operating at quadrature can now determined.

The mean-square shot-noise current is:

$$\left\langle i_n^2 \right\rangle = 2eB\left\langle i_D \right\rangle = \frac{2\eta e^2 B}{h\nu_{opt}}\left\langle P_D \right\rangle = \frac{2\eta e^2 B}{h\nu_{opt}} P_{tot}$$

where we note that $\left\langle \sin\omega_u t \right\rangle = 0$ (the time-average of a sine is zero).

The mean-square signal current is due to the second term in (QMI), and is given by:

$$\langle i_{sig}^2 \rangle = \left(\frac{\eta e}{h\nu_{opt}}\right)^2 \langle P_{sig}^2 \rangle = \left(\frac{\eta e}{h\nu_{opt}}\right)^2 \left\langle \left(2 k_{opt} M P_{tot} \delta \sin \omega_u t\right)^2 \right\rangle$$

$$= \left(\frac{\eta e}{h\nu_{opt}}\right)^2 \left(2 k_{opt} M P_{tot} \delta\right)^2 . \frac{1}{2}$$

where we note that $\left\langle \left(\sin \omega_u t\right)^2 \right\rangle = 1/2$

- Therefore the SNR of the Michelson interferometer is given by:

$$SNR = 2 k_{opt} M \delta \sqrt{\frac{\eta P_{tot}}{4 h\nu_{opt} B}} \qquad \text{(SNR-M)}$$

The above can be used to define a figure-of-merit for the interferometer:

$$FOM = \frac{\delta_{\min}}{\sqrt{B/P_{tot}}} = \frac{1}{k_{opt} M} \sqrt{\frac{h\nu_{opt}}{\eta}} \qquad \text{units: meter.}\sqrt{W}/\sqrt{Hz} \qquad \text{(MDD-M)}$$

which gives the minimum detectable displacement for unit bandwidth and unit total optical power incident on the photodetector.

Q: What ratio of object beam to reference beam intensities (at the photodetector) leads to the best SNR?

2.5.2 <u>Heterodyne Michelson interferometer</u>:

This is again a two-beam interferometer, but in this case, the two beams to be mixed are of slightly different optical frequencies. Typically this is obtained by passing a laser beam through an acousto-optical modulator (Bragg cell) as shown in Fig. 2.4.

The Bragg cell consists of an optical crystal with a piezoelectric transducer attached to it. The pzt-transducer is excited continuously to produce a harmonic ultrasonic wave of frequency ω_B. The construction of the Bragg cell is such that this leads to a standing ultrasonic wave in the crystal, which results in a spatially periodic variation of the stress-strain state in the crystal. This in turn (through the elasto-optic effect) results in a spatially periodic variation in the optical refractive index of the material. That is, a standing refractive index grating is induced by the pzt-transducer, which causes the incident optical beam to diffract. In what is known as the Bragg-regime of diffraction, typically only a zeroth-order and a first order beam are obtained. The zeroth order beam is essentially the undiffracted beam, and the first-order diffracted beam comes off at an angle and is frequency-shifted by ω_B.

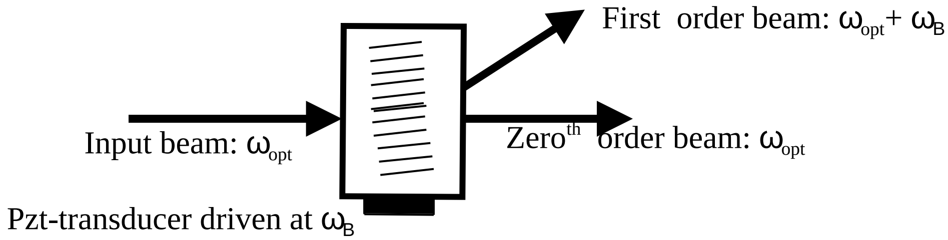

Figure 2.4: A Bragg-cell (acousto-optic modulator)

Suppose that the two output beams from an AOM are used in our two-beam interferometer setup, with say the frequency-shifted beam being used as the reference, and the undiffracted beam being used as the object beam. Upon re-mixing these beams at the photodetector, we have:

$$E_R = a_R e^{-j(\phi_R - (\omega_{opt} + \omega_B)t)}$$
$$E_o = a_o e^{-j(\phi_O - \omega_{opt} t)}$$

The resulting interference intensity is:

$$P_D = P_R + P_O + 2\sqrt{P_R P_O}\cos\{\omega_B t + (\phi_R - \phi_O)\}$$

Once again decomposing the phase difference in terms of a term $\varphi_n(t)$ that includes the effect of low-frequency noise as well as static path difference, and a signal term due to object displacement (assuming sensitivity to normal displacement), we have:

$$P_D = P_R + P_O + 2\sqrt{P_R P_O}\cos\{\omega_B t + \varphi_n(t) - 2k_{opt}\delta(t)\}$$

Once again, considering a sinusoidal object displacement which is a fraction of the optical wavelength $\delta(t) = \delta \sin\omega_u t,\ 2k_{opt}\delta << 1$, we have:

$$P_D = P_{tot}\left[1 + M\left\{\cos\left(\omega_B t + \varphi_n(t) - 2k_{opt}\delta\sin\omega_u t\right)\right\}\right]$$
$$= P_{tot}\left[1 + M\left\{\cos\left(\omega_B t + \varphi_n(t)\right)\cos\left(2k_{opt}\delta\sin\omega_u t\right) + \sin\left(\omega_B t + \varphi_n(t)\right)\sin\left(2k_{opt}\delta\sin\omega_u t\right)\right\}\right]$$
$$= P_{tot}\left[1 + M\left\{\cos\left(\omega_B t + \varphi_n(t)\right) - 2k_{opt}\delta\sin\omega_u t.\sin\left(\omega_B t + \varphi_n(t)\right)\right\}\right]$$
$$= P_{tot}\left[1 + M\left\{\cos\left(\omega_B t + \varphi_n(t)\right) - 2k_{opt}\delta\left[\underbrace{\cos\left((\omega_B + \omega_u)t + \varphi_n(t)\right)}_{\text{signal at upper side-band}} - \underbrace{\cos\left((\omega_B - \omega_u)t + \varphi_n(t)\right)}_{\text{signal at lower side-band}}\right]\right\}\right]$$

Note that the object displacement information can be directly monitored on a spectrum analyzer and is proportional to the amplitude of the signals at the upper and lower sidebands around the optical heterodyne frequency . Alternatively the signal can be extracted using an electronic demodulator as shown in Fig.2.5.

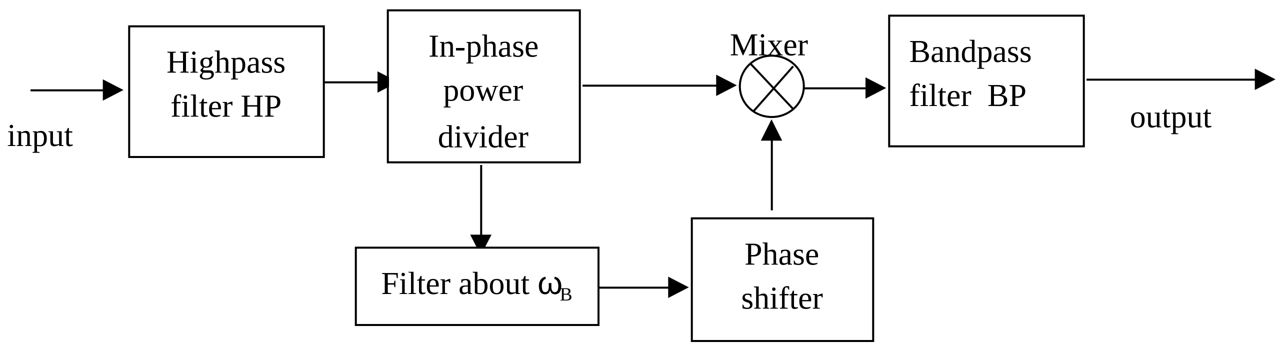

Figure 2.5: Phase demodulator

This demodulator does the following things:

- the HP filter filters out the low frequency (dc) components, letting through only: $\cos\left(\omega_B t + \varphi_n(t) - 2k_{opt}\delta\sin\omega_u t\right)$
- the in-phase power divider splits the signal into two parts
- the narrow filter about ω_B lets through only : $\cos\left(\omega_B t + \varphi_n(t)\right)$ which is essentially at the heterodyne frequency (along with the noise phase),

- the phase-shifter (this could be just an appropriate length of wire) converts this to a local oscillator signal: $\sin\left(\omega_B t + \varphi_n(t)\right)$

- the mixer multiplies the local oscillator signal with the input signal to create an output voltage:

$$\sim \left\{\cos\left(\omega_B t + \varphi_n(t) - 2k_{opt}\delta\sin\omega_u t\right)\right\}.\sin\left(\omega_B t + \varphi_n(t)\right)$$
$$\sim \sin\left(2\omega_B t + 2\varphi_n(t) - 2k_{opt}\delta\sin\omega_u t\right) + \sin\left(2k_{opt}\delta\sin\omega_u t\right)$$
$$\sim \sin\left(2\omega_B t + 2\varphi_n(t) - 2k_{opt}\delta\sin\omega_u t\right) + 2k_{opt}\delta\sin\omega_u t$$

- the bandpass filter BP then picks out only the unambiguous signal term by blocking the part at $2\omega_B$, letting out only:

$$V_{out}(t) \sim 2k_{opt}\delta(t)$$

That is, the output of the electronic demodulator is proportional to the ultrasonic displacement. There are several other ways to demodulate the heterodyne signal.

2.5.3 Operation on Rough Surfaces:

The problem with reference beam interferometers is that their performance on rough objects that scatter light is quite poor. If the object is not mirror reflective, the light impinging on it will be scattered, and we can no longer treat the interference as being between two plane waves. This results in degradation of the performance of reference-beam optical interferometers in two ways, as shown in Fig. 2.6. First, even the best lens system can only collect a fraction of the light that is scattered. From the shot-noise limited SNR expressions for the reference-beam interferometers, it is clear that this will result in loss of SNR.

Secondly, it is impossible to convert the scattered object beam back into a truly planar wavefront, and therefore the object beam will always be "speckled". Mixing a speckled wavefront with a planar wavefront is not efficient and indeed could be counterproductive with possible complete signal cancellation occurring. Look back at Fig. 2.3, and note that if different spatial points of the two beams being mixed are at different relative phases, complete signal cancellation can occur. For the homodyne interferometer, for example:

$$E_O(x,y) = a_O(x,y)e^{-j(\phi_O(x,y,t)-\omega_{opt}t)}$$
$$E_R = a_R e^{-j(\phi_R - \omega_{opt}t)}$$

and the *total* optical power in the photodetector is then:

$$P_D = \int_A \left[P_R + P_O(x,y) + 2\sqrt{P_R P_O(x,y)}\cos\left\{\phi_R - \phi_O(x,y) - 2k_{opt}\delta(t)\right\}\right] dA$$

where once again, I have split the object beam phase into "static+noise" and signal terms. (Note that earlier we did not explicitly integrate spatially because we were concerned with plane waves, and therefore the total optical power was just the power density times the area of the beams.) In view of the possible signal cancellation, typically one uses just "one speckle"; that is, we use only that portion of the light beam collected back with relatively uniform phase over the mixing plane. For these reasons, the performance of reference-beam interferometers is quite limited on rough surfaces. For example, the performance of the Michelson interferometer on optically-reflective specimens can be as high as U_{min} ~ 3pm (using P_T= 2mW HeNe; Δf = 10MHz; M=1; η = 0.75). This usually drops to about U_{min} ~ 1nm on typical rough surfaces.

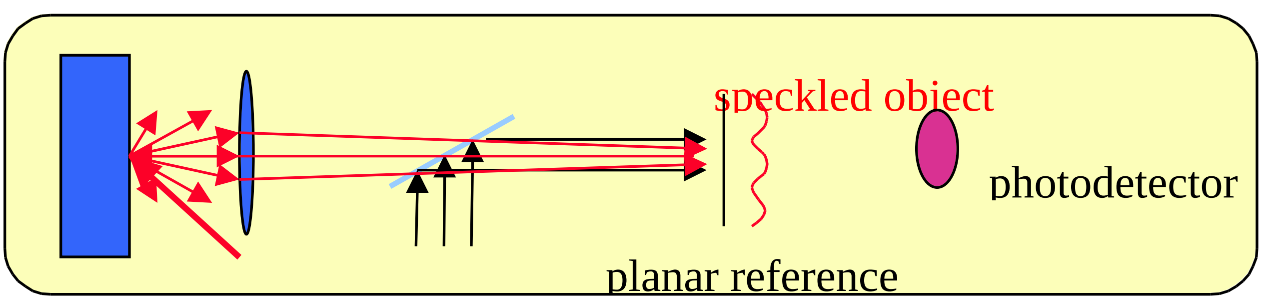

Figure: 2.6: Mixing of speckled and planar reference beams.

2.6 SELF-REFERENTIAL INTERFEROMETERS:

On rough surfaces, self-referential interferometers offer significantly improved performance. Here, the speckled object beam containing information about the object displacement is mixed with a *wavefront-matched reference* beam which may or may not contain signal information. Since the two beams are now wavefront-matched, it is possible to efficiently spatially mix the two beams to pull out the signal of interest.

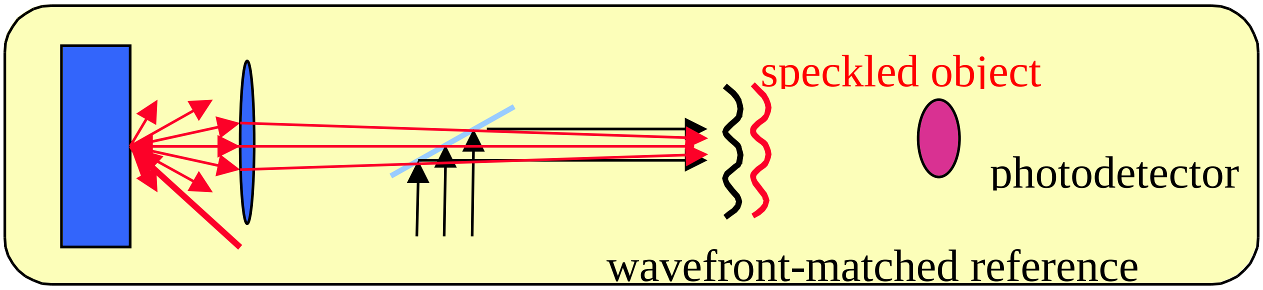

Figure: 2.7: Wavefront-matched interference.

Some of the common self-referential interferometers are: (i) time-delay interferometers, (ii) Fabry-Perot interferometers, (iii) adaptive holographic interferometers.

2.6.1 <u>Time-delay interferometers:</u> Consider the long-path time-delay interferometer shown in Fig. 2.8. Here, the scattered beam from the object is collected and split into two copies. Both these copies are reflected back by mirrors, but one of them travels via a short-leg and the other travels via a long-leg. Here both the beams that interfere are essentially wavefront-matched (assuming that the path difference is not so large that beam spreading becomes an issue), and so the spatial-mixing will be efficient. Here, both beams also possibly contain the signal of interest. So what does this interferometer measure? This will be a homework problem!

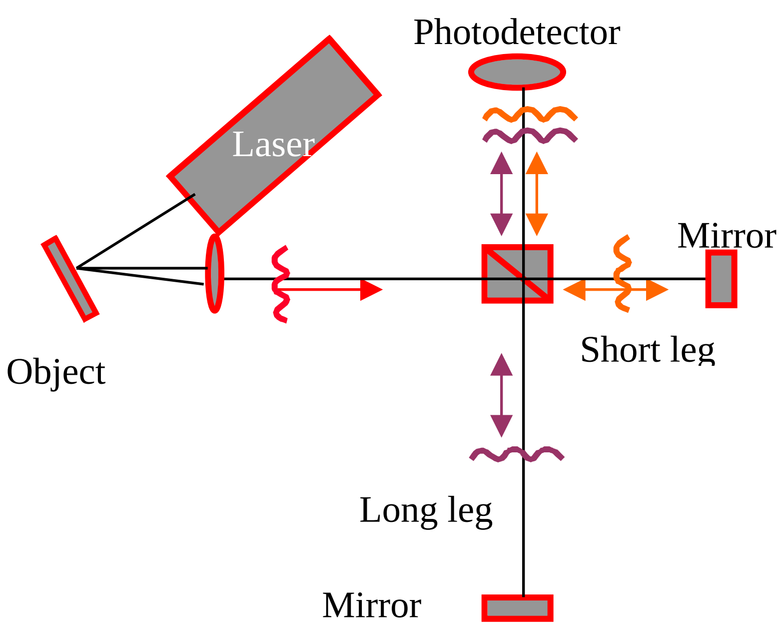

Figure 2.8: Long-path time-delay interferometer