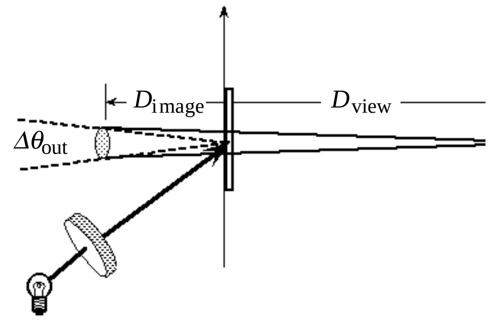
# Chapter 12: Phase Conjugation and Real Image Projection

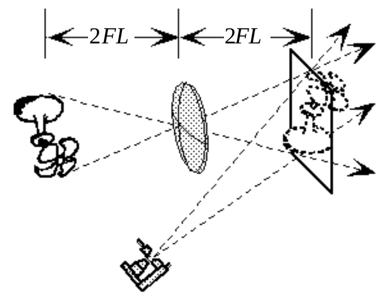



In the last chapter, we saw that having an image be much closer to the plate–compared to the viewer's distance–allows the use of larger-area and wider-band sources (that is, brighter sources) without blurring the image noticeably. But this is not as simple as just putting the object closer to the hologram plate—there it will usually block parts of the reference beam! And it is difficult to arrange for attractive object illumination if things get too close to the plate. Thus there is a lot of interest in techniques for optically "relaying" an image of a remote object, and then letting that relayed image serve as the subject of the hologram. Here, we will look briefly at some "conventional" techniques for image relaying, also called *real image projection*, and then concentrate on the more widely used holographic method.

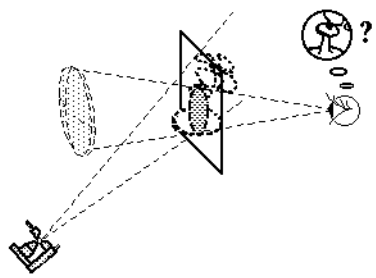
**Real-image projection techniques**

positive lens

A positive lens can form either a real or a virtual image of a scene. Here, we will consider the 2*F*:2*F* geometry, in which the lens forms a real image, same-sized but upside down, on the right-hand side. There, it can serve as an "optical object" for the hologram. We let the image straddle the hologram plane, with half of its depth on one side and half on the other, minimizing the maximum distance of the any part of the image from the hologram plate.




A major weakness of this approach is *vignetting*, a porthole effect on what the viewer sees that the lens causes because of its limited width. Because the viewer can see only those parts of the image that have open lens area behind them, only the central area of the image appears! As the viewer moves from side to side or up to down, different parts of the image become visible as they are "back lit" by the lens. Getting around this problem requires using lenses that are much larger than the object, perhaps twice its width, which become very expensive (if even practical—the example shown here is already an *f*/1 lens, which is very unusual!).

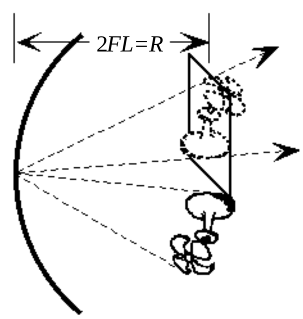



A secondary effect is that the image becomes distorted due to non-uniform magnification for those object parts not exactly 2*F* from the lens.

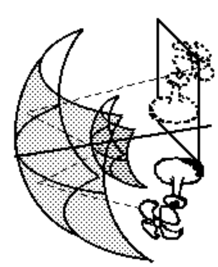
More complex setups use a second lens at the hologram plane to overcome the vignetting and non-uniform magnification, but then it is the viewing area that becomes limited[1]. The "bottom line" is that very few holo-cameras have been built that use lenses as the main imaging elements.

concave mirror

Much larger diameter-to-focal-length ratios are possible with concave mirrors, of which only spherical mirrors are really practical to fabricate. Most science museums have displays of real images produced by such mirrors, usually as part of a magical illusion (at the Boston Museum of Science, "space money" is floating upside down in a spinning goblet). Vignetting is not so much of a problem in this case, but the non-uniform magnification becomes even worse at large viewing angles.

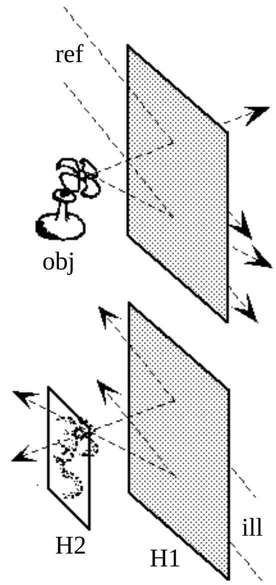



Multiple mirror systems have been described that correct for many of these distortions, but no such systems have yet found their way into practical use[2].



two-step holography

Holograms have slowly emerged as the optics of choice for real image projection, taking advantage of their conjugate-image projection properties. They can be made almost arbitrarily wide relative to the object, thus affording an unvignetted image over quite a wide angle of view. Of course, this becomes a non-real-time or two-step process: the first hologram has to be exposed and processed, and only then illuminated to provide the image for the second or final hologram. With two holograms to deal with, we have to adopt a naming strategy to avoid confusion. The first hologram is often called the "master" hologram or the "H1." The second hologram is generally called the "transfer" hologram or the "H2." We will arbitrarily adopt H1 and H2 as the designators for most of our discussions here.



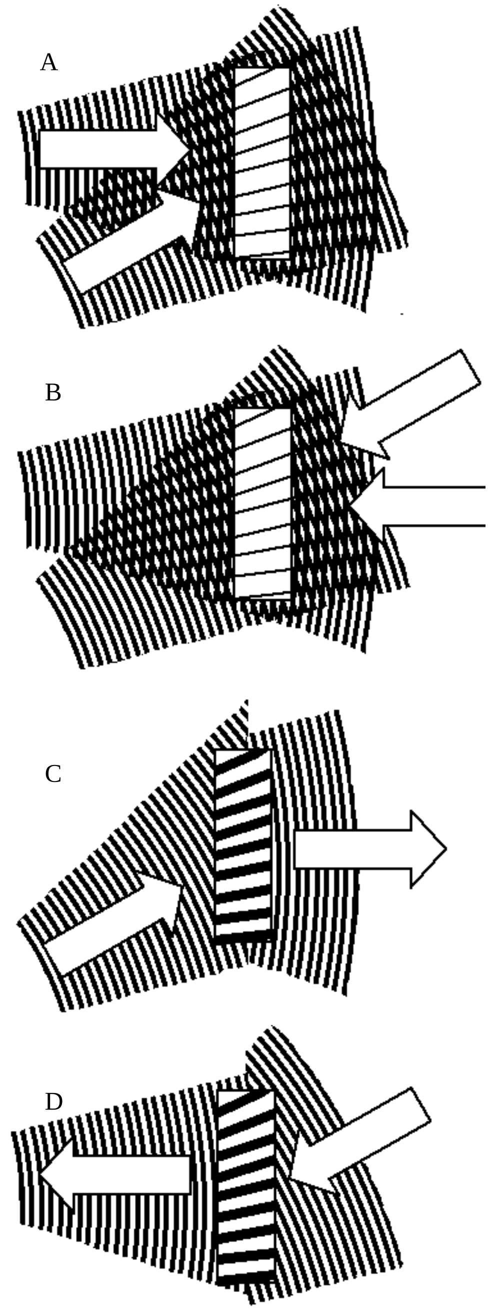

The second sketch shows the H1 being used in a way that is brand new for us—it is being illuminated through its back! The image it produces presents a real image to the H2 all right, and it is properly described as an $m$=–1 or *conjugate* diffracted order, but it has several properties that we will have to explore fairly carefully. This chapter will concentrate on this new type of conjugate image projection before going ahead to consider the resulting H2 and its properties.

**Phase conjugation—a descriptive approach**

Holograms have a property that no other optical device has: the ability to generate a so-called "phase conjugate" image, one that behaves as though the waves from an object were literally traveling backwards in time to generate an image of that object focused in space. The image is called "phase conjugate" because the sign of the phase of its wavefront, as generated by the hologram, is exactly the opposite of that of the "true" or virtual image wave. It is the same conjugate reconstruction term that we associate with the $m$=–1 order, except that the illumination is now traveling in the direction opposite (typically right-to-left) to the reference beam. Other descriptions of this kind of reconstruction are "reverse ray tracing," "time-reverse waves," and similar-sounding terms. Optical devices such as retroreflectors ("Scotchlite" for example) approximately conjugate the wave from a point source, sending the light roughly back in the direction that it came from. Photorefractive and non-linear optical materials can be used in the four-wave mixing mode, which is also called "real-time holography," which produces an exactly phase-conjugated wavefront. But in this course we will limit our attention to the two-step holographic, recording & reconstruction, type of phase conjugation.

thick hologram—general conceptual approach

We can understand the central concepts of wavefront phase conjugation with a fairly simple geometrical example. These are concepts that apply to thick as well as thin holograms, and this “proof” will include both cases. The basic idea is that the exposure of a holographic plate is a summation of energy over time, and that once the exposure is finished, the plate has no idea of whether time was running forward or backward. Consider holograms made in two different waves: by two diverging waves traveling from the left, as in Fig. A, and by two converging waves traveling from the right, as in Fig. B. The curvatures and angles of the two reference waves (and also the object waves) are the same, but the waves are traveling in opposite directions, as though a high-speed movie of the A waves were being played backward in B. That is, reference beam B is a converging beam, focused to the location of the point source for reference beam A (and object beam B is similarly focused to location of the object point source A). The identification of the reference and object beams is left deliberately ambiguous, as the result doesn’t depend on which is which, but you might think of the upper arrow in A as the object beam, and the lower as the reference beam.




It might already be clear that the holograms/gratings produced in these two cases are identical! The exposure doesn’t distinguish between waves traveling toward the right and toward the left. But let’s keep each with its intended illumination for just a minute more.

Certainly, if we illuminate the hologram from A, which we will call $H_A$ in Fig. C, with a replica of its reference beam, it will reconstruct a perfect replica of the diverging object wave, producing a virtual image of a point at the location of the source for A. Likewise, if the hologram from B, which we will call $H_B$, is illuminated with a replica of its reference wave as in Fig. D, it will reconstruct a perfect replica of its converging object wave, producing a real image of a point, also at the location of the source for A.

Now, the trick is to switch the two holograms, $H_A$ and $H_B$, while nobody is looking. Because they are identical, no one will be able to tell the difference—each will reconstruct perfectly in the others intended illumination! That is, a hologram of a point source ($H_A$) can produce a real image of a point simply by illuminating it through its back with a wave that has a particular relationship to the original reference wave—it must be its *phase conjugate* (it has the same shape, but is traveling as though reversed in time).

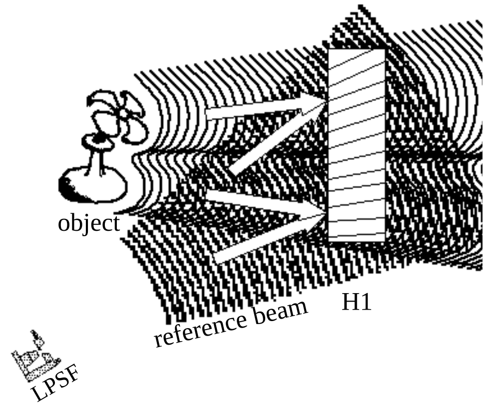
This generalizes to waveforms of arbitrary shape, as long as the amplitude of the reference and illumination waves are uniform so as to produce an accurate replica of the object waves. No matter how complex the shape of the object waves, for example. Thus, this principle applies to light diffusely reflected by a solid three-dimensional object, among many other things. For complex objects, which can be considered as collections of points arrayed in space, it already becomes clear that the image produced by perfect phase conjugate illumination is also three-dimensional, perfectly undistorted and projected into space, but with some peculiar properties that we will explore in just a minute.

**Perfect conjugate illumination (examples):**

The accuracy of the 3-D reconstruction depends on the accuracy of the phase conjugated illumination, measured with respect to the reference wave. Thus we will briefly examine some practical implications of a few examples before continuing.

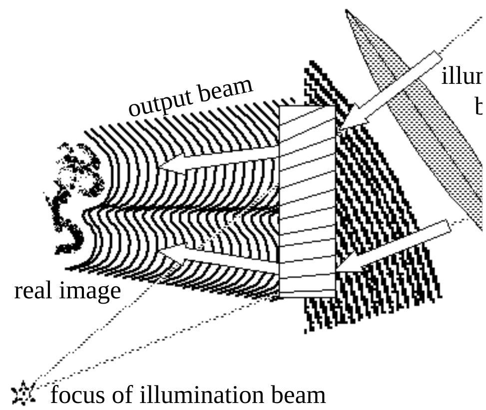
diverging/converging

The first example shows a diverging reference beam, for which the phase conjugate is a converging reference beam, focused at the origin point of the reference beam. This illumination beam has to be converged by some optical device, typically a positive lens or a concave mirror, which has to produce an accurate point focus, without any aberrations. Note that, in general, the optic for producing such a beam has to be significantly larger than the hologram it is intended to illuminate. Because the cost of optical elements typically grows as the third power of their maximum diameter (or faster!), lens size is an important economic consideration.



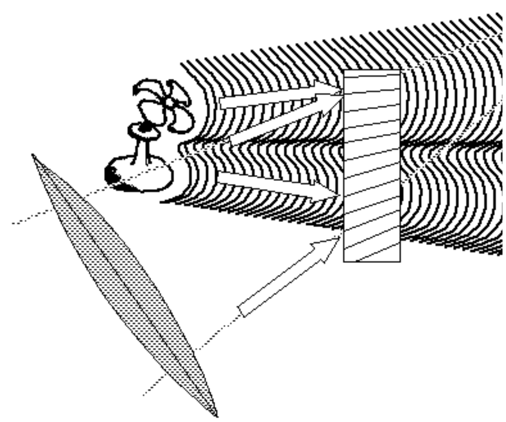

converging/diverging

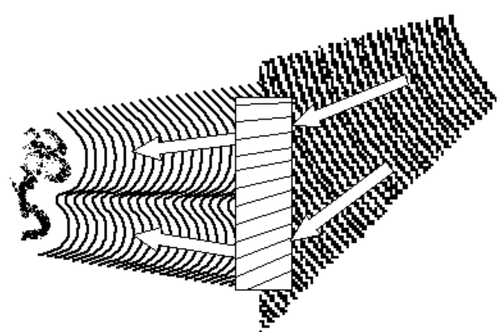
The converging optic may also be used for the reference beam, which is handy for illumination situations where there is no opportunity to use extra optical elements. However, very short illumination beams require impractically “fast” reference beam converging lenses, compared to virtual or direct image projection. The reference beam lens must be as close as possible to the hologram to keep its diameter to a minimum, which also makes the setup awkward.




plane/plane

The most generally useful configuration is one in which collimated light is used for both the reference and illumination beams. We should have qualms about using any optical element after the spatial filter in a reference beam, due to the magnification of the effects of dust and so forth, but given the lack of practical alternatives, this seems like a reasonable compromise. The collimators need be only as big as the hologram, although some extra size helps keep the beams free of “edge ringing” patterns, and they can be placed as far from the plate as is convenient, which helps simplify the exposure geometry. Later on we will make white-light viewable holograms, for which the sun is a handy illuminator and which produces collimated light, so that a collimated reference beam is actually an appropriate choice.

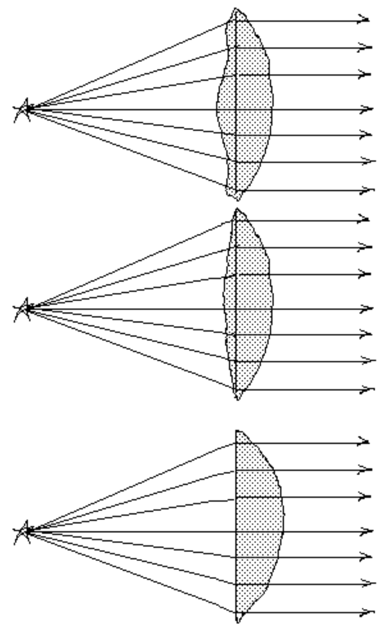




collimator choices

A holographer typically needs at least one collimator, and preferable two, for making high-quality holograms. They are inevitably large, expensive, heavy, and easily damaged items, which brings new aspects of thoughtful care to the laboratory. Holographic-grade collimators are not available "off the shelf" anywhere, and have to be custom made or adapted from available components. Let's stop for a moment's practical discussion of some of the options that confront this choice:

refractive collimators (lenses):



A fairly simple positive lens can produce a collimated beam with acceptable accuracy. Ideally, one surface of the lens should have an aspheric shape (called a Rubé lens), but non-spherical surfaces are incredibly expensive to make and test.

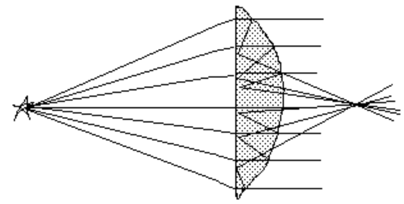
Your lens will doubtless have spherical surfaces, and thus the focal length at the edge will be slightly shorter than at the center, due to spherical aberration that is inherent in optics with spherical surfaces (hence the name). This aberration can be minimized by making the surface of the lens facing the point source much less curved than the other side, by "bending" the lens in effect. Using more than one element can further minimize the spherical aberration, and can also minimize the variation of focus with color, the chromatic aberration. But multiple surfaces can create serious multiple-reflection problems too, and even a single element can be very expensive in large sizes.

The optimum curvature of the lens surface facing the point source is about a tenth that of the other surface, but it is much cheaper to have a flat surface made than any spherical surface. There is very little glass to remove from the blank, and it is very easy to test (opticians usually have to make a testing element for every new surface curvature). Thus all of the curvature is usually put on one surface, the one facing away from the point source. The problem is that all of these compromises mean that the lens focal length has to be at least four times the diameter of the lens or the ray-pointing errors at the edge of the lens will become unacceptable. The exact criterion depends on the use the holograms will see, and the precision of the imaging. But limiting simple plano-convex collimators to $f/4$ or more (ratio of focal length to diameter) is a good rule of thumb.

Getting the overall surface curvature correct is something that most optical shops can do well. The only issue may be the maximum diameter of lens that their equipment can handle. However, variations of surface curvature and flatness over small distances are especially troublesome in laser applications. The glass surface often takes on a very shallow random waviness during polishing. When used as a collimator, the beam may look uniform at near distances, but will take on a mottled appearance a few meters downstream. This pattern resembles the roughness of the skin of an orange, and it is important to specify that there should be no "orange peel" on the lens surface.

Another issue is bubbles and straie in the glass. Normal specs for "bubble free" usually say "no bubbles bigger than a millimeter in diameter in the center third of the lens," or some such. Emphasize that you can tolerate no bubbles at all, which may limit your choices of glass types to the most popular varieties, such as BK-7. If the molten glass is improperly mixed before cooling, "ropes" of material of higher or lower index may form inside the glass. These produce "straie" (they look like streamers in the downstream light). This is often not a problem for conventional uses, but a disaster in a hologram reference beam. There are many front elements from big theatrical spotlights that would make wonderful collimators except for this usually-hidden defect!

A final issue is anti-reflection coatings.  Without treatment, each naked glass surface reflects about 4% of the incident light, the Fresnel reflection.  Enough light is doubly reflected to produce a point image two focal lengths in front of the collimator, called the Boys' image.  A single layer of an evaporated material such as magnesium fluoride that is exactly a quarter of a wavelength thick can reduce the Fresnel reflection to under 1%.  But because the Boys' image concentrates the light, it can still be objectionably strong, and more elaborate coatings are usually needed.  A three-layer coating can be designed to completely eliminate the reflection at a single wavelength (a "V-coat") or to reduce it to about 0.25% over the entire visible spectrum (a "BBAR-coat").  The choice is difficult because one always wants to keep open the option for full-color holography, even with the chromatic aberration of the lens in mind.  Most of our lab lenses have BBAR coatings, and we move the collimator far enough away from the hologram to attenuate the effect of the weakened Boys' image.

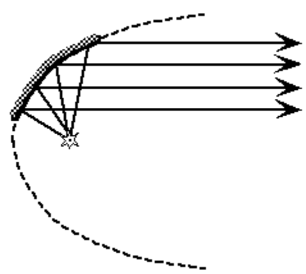


Bottom line, a finished and coated glass collimator can be custom made for about $700 for a 7" diameter, and about $4000 for a 14" diameter (these are obsolete prices from Harold Johnson Optical Labs).  If you can find a group of friends to share the cost of a run of four or six lenses, the price will come down quite a bit.

People are occasionally tempted to have collimators made in acrylic plastic. There are a few shops around who do very good work in plastic, and it grinds so much more quickly than glass that there are huge cost savings.  However, it is very difficult to avoid orange peel in polishing plastics (probably impossible!), and no durable anti-reflection coatings are available (they have been compared to cake frosting by one dismayed holographer).  And of course acrylic is soft and incredibly vulnerable to accidental damage during use.

reflective

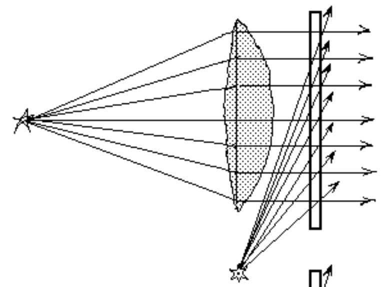
A realistic alternative to a collimating lens is a collimating mirror.  Telescope mirrors, after all, produce point images of distant stars—the inverse of collimation!  An ideal collimating mirror would have a parabolic shape, as most large telescope mirrors do.  However, conventional telescopes put the pickup optics along the axis of the mirror, where they block the very center of the beam. This is not acceptable in holography, so the ideal mirror would be an off-axis section of a parabola.  Unfortunately, only rotationally symmetric mirrors can be made with high accuracy, so a very large parabola would have to be generated, and one-sixth of it cut out for your use—but you would have to pay for all six sections!

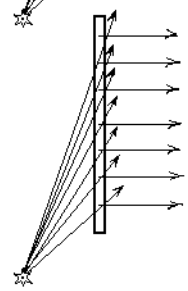


Instead, holographers use spherical mirrors tipped off of axis, and try to correct the resulting astigmatism by feeding the mirror through a lens that is slightly tipped to produce the opposite astigmatism.  Fairly cheap spherical mirrors are available from astronomy suppliers (a 12" dia. mirror is $800 from Coulter Optical), but typically have focal lengths at least ten times their diameters, which makes for setups that spread out over large distances.

Orange peel is also an issue for mirrors, and scratches and digs take the place of internal bubbles.  Straie are not a problem, though, and neither is chromatic aberration.  Holographers tend to be strongly partisan in their preference for refractive or reflective collimators, so be careful who you ask about which type!

diffractive



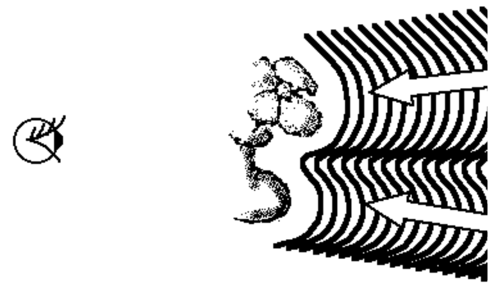


A third alternative would seem to be an ideal choice, but is not yet commercially available: a holographic collimator. If someone who had invested in a good collimator were willing to make holographic "clones" of it, they would find a ready market, even at several hundred dollars per hologram. Simply exposing a plate to a collimated beam and a steeply diverging beam, and mounting it on flat anti-reflective-coated glass would seem to produce exactly what is needed. However, such diffractive optical elements are the most sensitive types of holograms to exposure and/or processing flaws, and collimators of useful quality are incredibly difficult to make. Nevertheless, I am confident that someday some holo-entrepreneur will decide to take on this selfless task, and help put this important tool into the hands of even small-scale holographers.

**Pseudoscopic image properties (ideal):**

Now we return to more discussion of the peculiar real image that is projected by a phase-conjugate illumination beam. From the vantage point of an observer, who is down-stream in the now leftward-propagating optical beam, it presents an appearance never seen before!

<u>outside-in</u>



**pseudoscopic image**

Note that the hologram can only reproduce information about parts of the object that it has "seen," which are those parts closest to the hologram plate (which obscure its view of the more distant parts). Thus only the "front" surface of the object (or right-hand surface in this sketch) is reproduced in space as a glowing texture—the back of the object simply does not appear. The observer is then seeing the front of the object through the back—the occlusion cues are just the opposite of what they should be! With practice, an observer can learn to see this image as an "outside-in" version of the object. A ball will appear as a cup, for example. The apparent depth of the object has been reversed with respect to its occlusion cues, as they would be for a stereoscope image if the right and left views were reversed. The descriptor for this reversed-depth type of image is *pseudostereoscopic*, or more commonly, *pseudoscopic*. For a naive observer and a complex object, though, the occlusion cues usually dominate the parallax cues, and the image is seen as having normal depth but also as rotating as the observer moves from side to side.

**Effects of imperfect conjugates - descriptive:**

Nothing in our conceptual discussion prepares us to describe what happens when the illumination departs from being the <u>perfect</u> phase conjugate of the reference beam. For that, we need to develop some mathematical models to describe the behavior of the light waves. Also, we need to give up our ability to describe really thick holograms. As the local illumination angle rotates from that required for perfect phase conjugation, the output beam angle rotates in the same direction (although by a somewhat different amount, owing to the non-linearity of the sine function), and the amplitude of the beam decreases because the incoming and outgoing beams no longer satisfy the Bragg conditions for volume diffraction. We will assume that the hologram is thin enough that the Bragg angle mismatch problems are not very severe.

If we start by assuming that the perfect phase-conjugate illumination would be collimated, then reconstruction by a diverging illumination with the same central angle means that although the center of the plate is illuminated at the proper angle, the illumination at the top of the plate is rotated slightly to the right, and the illumination at the bottom of the plate is rotated slightly to the left. The output rays from the top and bottom are then also rotated slightly to right and left respectively, and cross the undeviated ray from the center somewhat further from the hologram than before. Thus the image distance will become greater as the radius of curvature of the illumination wave becomes shorter.

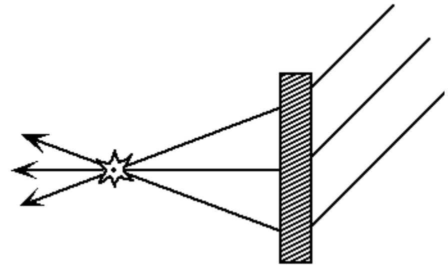
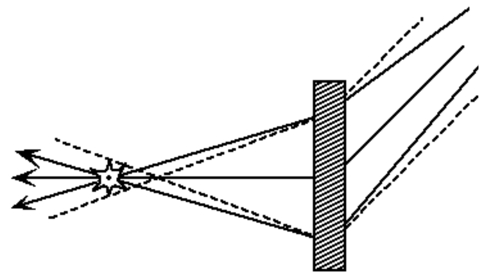


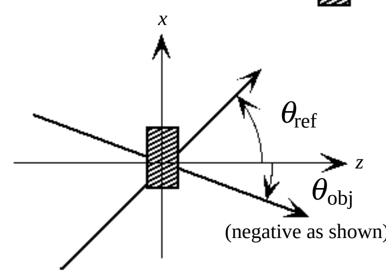
**Image location (analytical):**

The relationship between the illumination and output angles is described by exactly the same equation that we have seen before, with the definition of angles increased to explicitly include angles greater than 90°, which must be measured “the long way around” from the plate perpendicular (which continues to be defined as coming out of the side of the plate <u>opposite</u> to that exposed to the <u>object</u> beam). Namely, the “sin$\theta$” equation, Eq. 3 of Chap. 8 still applies,



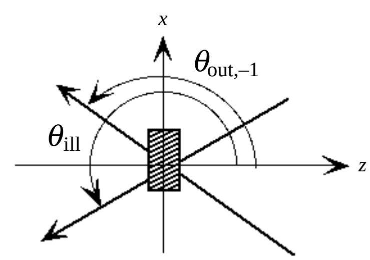
$$\sin\theta_{\text{out},m} = m\frac{\lambda_2}{\lambda_1}\left(\sin\theta_{\text{obj}} - \sin\theta_{\text{ref}}\right) + \sin\theta_{\text{ill}} \ . \qquad (1)$$

And if we apply this equation over a small area and note where the output rays intersect (within the paraxial approximation), we find that the same focus-law relationships, the “1/R” and “cos-squared” equations (Eqs. 7 and 8 of Ch. 10), also apply to this case:




$$\frac{\cos^2\theta_{\text{out},m}}{R_{\text{out},m,x}} = m\frac{\lambda_2}{\lambda_1}\left(\frac{\cos^2\theta_{\text{obj}}}{R_{\text{obj}}} - \frac{\cos^2\theta_{\text{ref}}}{R_{\text{ref}}}\right) + \frac{\cos^2\theta_{\text{ill}}}{R_{\text{ill}}} \ \text{, and} \qquad (2)$$

$$\frac{1}{R_{\text{out},m,y}} = m\frac{\lambda_2}{\lambda_1}\left(\frac{1}{R_{\text{obj}}} - \frac{1}{R_{\text{ref}}}\right) + \frac{1}{R_{\text{ill}}} \ . \qquad (3)$$




In other words, the same equations apply regardless of what direction the light is traveling in, as long as we are careful to define the angles and distances properly, especially by identifying converging waves with negative radii of curvature. In addition, the diffracted order of interest in phase conjugation is almost always the $m$ = -1 order.

If we imagine that the holograms are perfectly thin, then the illumination and the output waves have the same phase patterns whether they are traveling from left or right, and the transmittance of the hologram operates the same way in both directions. Our preference for now using leftward-going illumination is to better match the angle of thick holograms for bright reconstructions in the $m$ = -1 order, and to make certain that the image reads properly top to bottom and left to right. Otherwise, it is important to realize that this is not a new hologram output term. It is the same conjugate term that we have seen before, except that the illumination has been angled to make it more accessible.

**Image magnification:**

Because the sin$\theta$ equation still applies in phase conjugate reconstruction, all of our previous image formulae also apply—because they all followed from the application of the sin$\theta$ equation in the direct or forward reconstruction context. Thus the magnification formulae still apply, again provided only that the usual case is $m$ = -1 and all angles and distances/radii of curvature are measured correspondingly. In general, because the most usual case of imperfect conjugation is using an illumination wave that is diverging more than it should, the output waves will be diverging more than they should, and the image will focused further from the hologram than it should be.

For the image of an extended object, consisting of many points, the rays through the center of the hologram will be traveling in the same directions as for perfect conjugation, but the rays from the margins of the hologram will again cross the central rays further away from the hologram. The real image will therefore be magnified in the ratio of exposure to reconstruction distances as for the previous cases, and it will also suffer the same effects of change of the wavelength of the illumination.

longitudinal magnifications (astigmatic):

$$MAG_{\text{long},x} = \frac{\Delta R_{\text{out},m,x}}{\Delta R_{\text{obj}}} = m\frac{\lambda_2}{\lambda_1}\left(\frac{R_{\text{out},m,x}}{R_{\text{obj}}}\right)^2 ,$$
$$MAG_{\text{long},y} = \frac{\Delta R_{\text{out},m,y}}{\Delta R_{\text{obj}}} = m\frac{\lambda_2}{\lambda_1}\left(\frac{R_{\text{out},m,y}}{R_{\text{obj}}}\right)^2 . \qquad (4)$$

lateral magnification (astigmatic):

$$MAG_{\text{lateral},x} = m\frac{\lambda_2}{\lambda_1}\frac{\cos\theta_{\text{obj}}}{\cos\theta_{\text{out},m}}\frac{R_{\text{out},m,x}}{R_{\text{obj}}} ,$$
$$MAG_{\text{lateral},y} = m\frac{\lambda_2}{\lambda_1}\frac{R_{\text{out},m,y}}{R_{\text{obj}}} . \qquad (5)$$

**Relation to the lens & prism-pair model:**

Recall the model of an off-axis transmission of a single point that consisted of a base-down prism plus a negative lens, and which had an accompanying base-up prism and positive lens (each prism of the same but opposite deflecting power, and each lens of the same but oppositely-signed focal length). It doesn't matter which element comes first in the beam, and rearranging slightly makes it easy to see how the "opposite set" (the conjugate order of the hologram) comes into play to deflect and focus the collimated illumination to produce a real-image focus at the location of the former virtual image. Each point of the object gives rise to such a pair of prism & lens sets, and thus gives rise to a corresponding real image point.

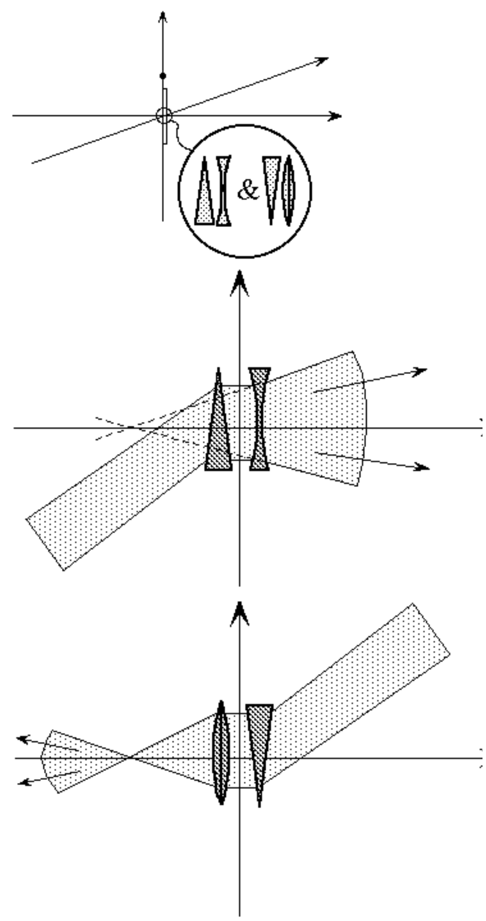
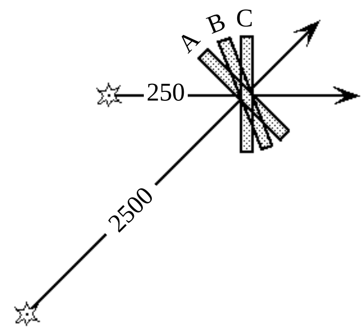
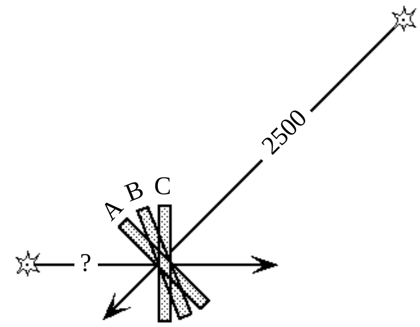


**Image aberrations-*astigmatism*:**

When $m$=-1, the reference and illumination terms of the focusing equations (the one-over-R and cos-squared equations) add, and so any departures from ideal phase-conjugate illumination also add. Most typically, both the reference and illumination beam are weakly diverging, causing the image to be formed farther from the plate than the object location, and thus magnified. In addition, the wavefronts are likely to be *astigmatic*, so that no sharp focus can be obtained. Here the trick of splitting the reference and object beam angles (and later, the image and illumination angles) with the perpendicular to the plate comes in especially handy, as astigmatism is balanced out in this case. The table below shows the vertical and horizontal focal distances for the three exposure and reconstruction plate angle shown in the sketch.




| | plate angle | | |
|---|---|---|---|
| | A | B | C |
| horizontal focus | 312.5 | 312.5 | 312.5 |
| vertical focus | 416.7 | 312.5 | 277.8 |

Of these, the two most common geometries are B, in which the plate normal/perpendicular bisects the object and reference beams, and for which there is no astigmatism (a very important consideration), and C, in which the object and output are perpendicular to the plate, and for which the vertical focus is always closer to the plate than the horizontal focus.

**Conclusion**

Real-image projection by phase conjugate illumination will turn out to be one of the most powerful techniques we use in holographic imaging. It will certainly dominate the second half of this course! There are only three things to remember to do: 1) measure illumination and output angles “the long way around” from the perpendicular to the “back” of the plate, 2) let the order number, $m$, be negative one, and 3) remember that negative radii of curvatures signify converging waves (producing real images). This marks the end of the convention that light will be traveling from left to right, and that the front of the plate will be facing left. From now on, we will expose plates from whatever direction is convenient, and reconstruct them after moving them around. The local coordinate system will have to follow the plate accurately, which can get pretty confusing! So make sure that you understand what is happening here before we start tumbling around in holographic space.

References:

1. S.A. Benton, H.S. Mingace, Jr., and W.R. Walter, “One-step white-light transmission holography,” SPIE Proc. Vol. #215, pp. 156-161 (1980).
2. Steele, W.H. and Freund, C.H., "Single-step rainbow holograms without distortion," Optics Communications #51, pp. 368-370 (1984).

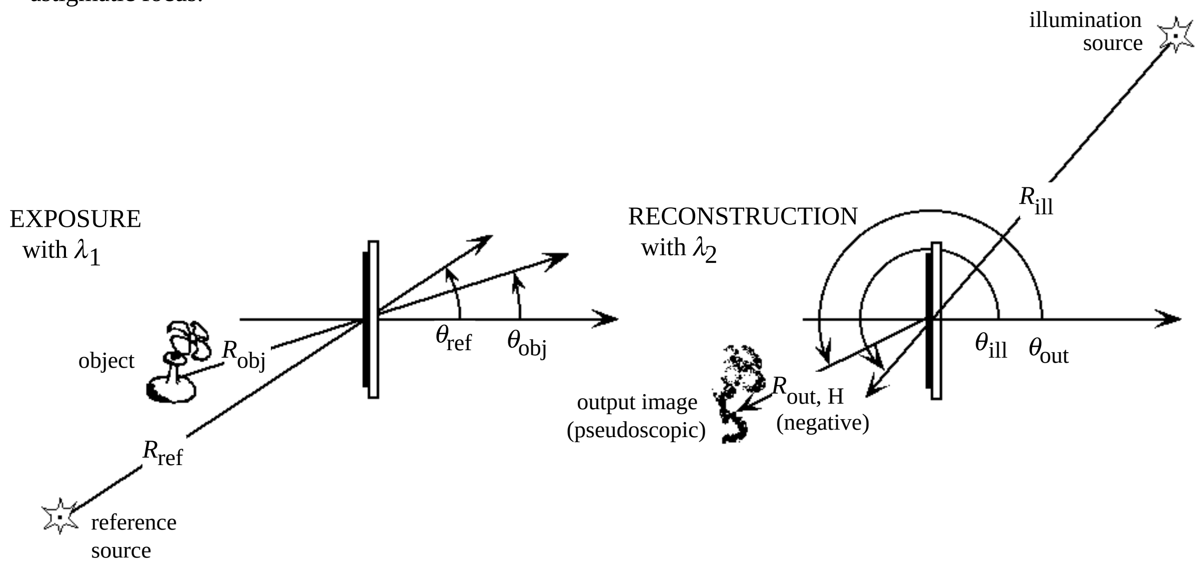
# Ø-C HORIZONTAL FOCUS: OFF-AXIS TRANSMISSION HOLOGRAPHY

"Phase-Conjugate" or "Time-Reverse" Reconstruction

Illumination angle ≈ reference angle + 180°, ***m* = –1**, usually producing a real image, so that $R_{out}$ is **negative** (a converging wave) and the image is "pseudoscopic."

<u>Horizontal</u> or "out-of-plane" focus:

Marginal rays are coming out of the page or *x*-*z* plane. Also known as the "*y*-focus," the "parallax focus," or the "sagittal astigmatic focus."

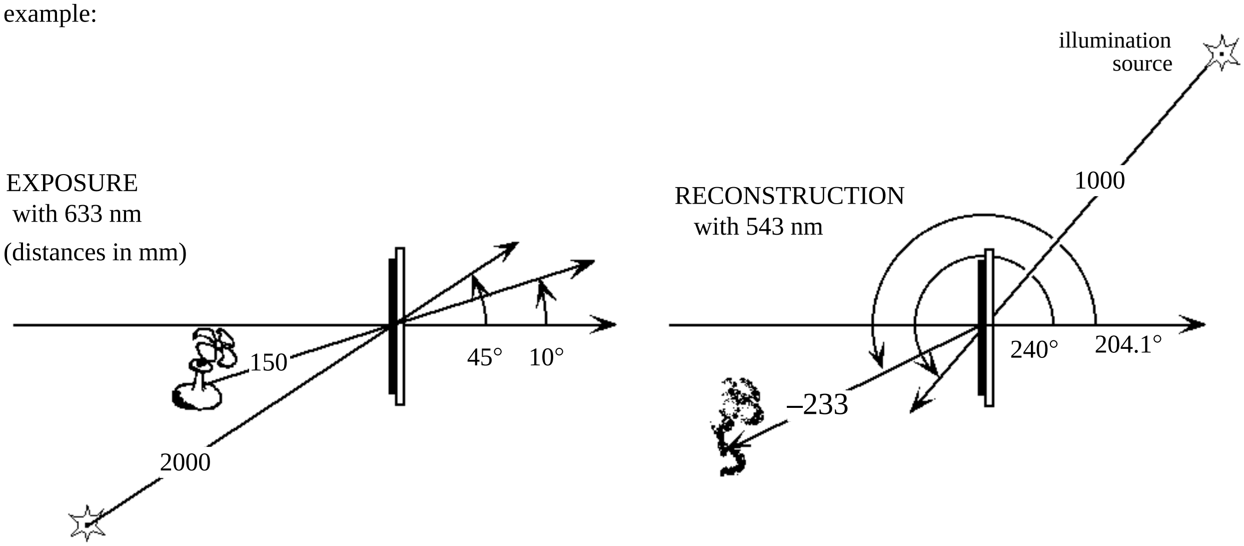



$$\frac{\sin\theta_{out,m} - \sin\theta_{ill}}{\lambda_2} = m\frac{\sin\theta_{obj} - \sin\theta_{ref}}{\lambda_1}, \qquad m = 0, \pm 1, \pm 2, \ldots$$

$$\frac{1}{\lambda_2}\left(\frac{1}{R_{out,m,\mathrm{H}}} - \frac{1}{R_{ill}}\right) = m\frac{1}{\lambda_1}\left(\frac{1}{R_{obj}} - \frac{1}{R_{ref}}\right)$$

Magnification - same equations as "direct, horizontal," but with $m = -1$.

example:




$MAG_{lat}$ = 133%, $MAG_{long}$ = 207%

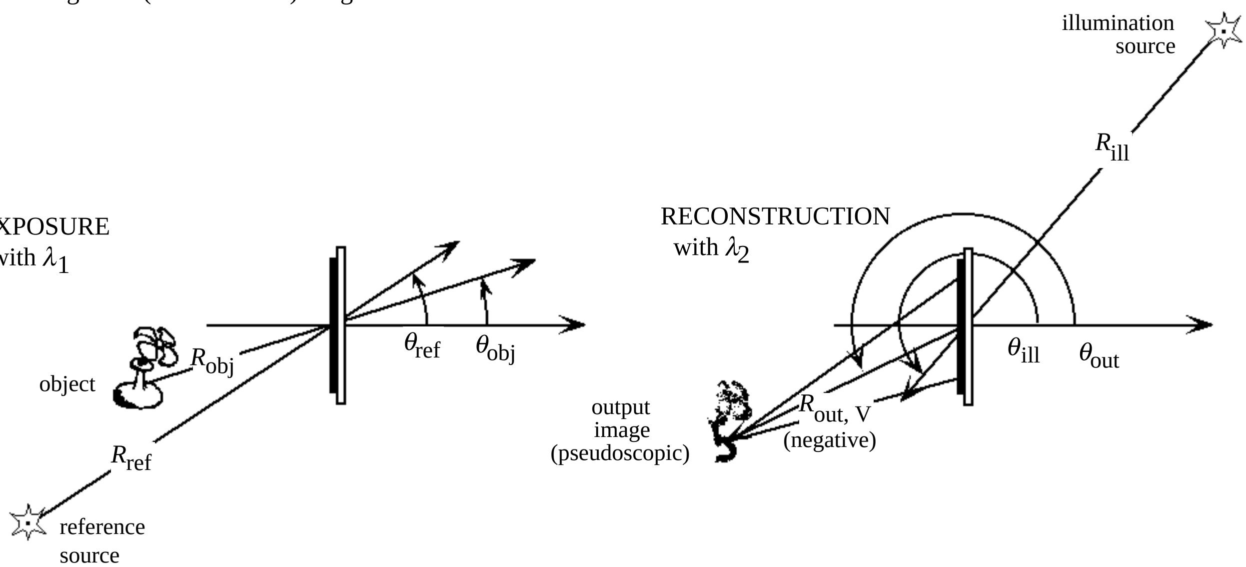
## **Ø-C VERTICAL FOCUS:** OFF-AXIS TRANSMISSION HOLOGRAPHY

"Phase-Conjugate" or "Time-Reverse" Reconstruction

Illumination angle ≈ reference angle + 180°, $\boldsymbol{m = -1}$, usually producing a real image, so that $R_{out}$ is **negative** (a converging wave) and the image is "pseudoscopic."

<u>Vertical</u> or "in-plane" focus:

Marginal rays are in the plane of the page, the *x-z* plane. Also known as the "*x*-focus," the "color focus," and the "tangential (or meridional) astigmatic focus."

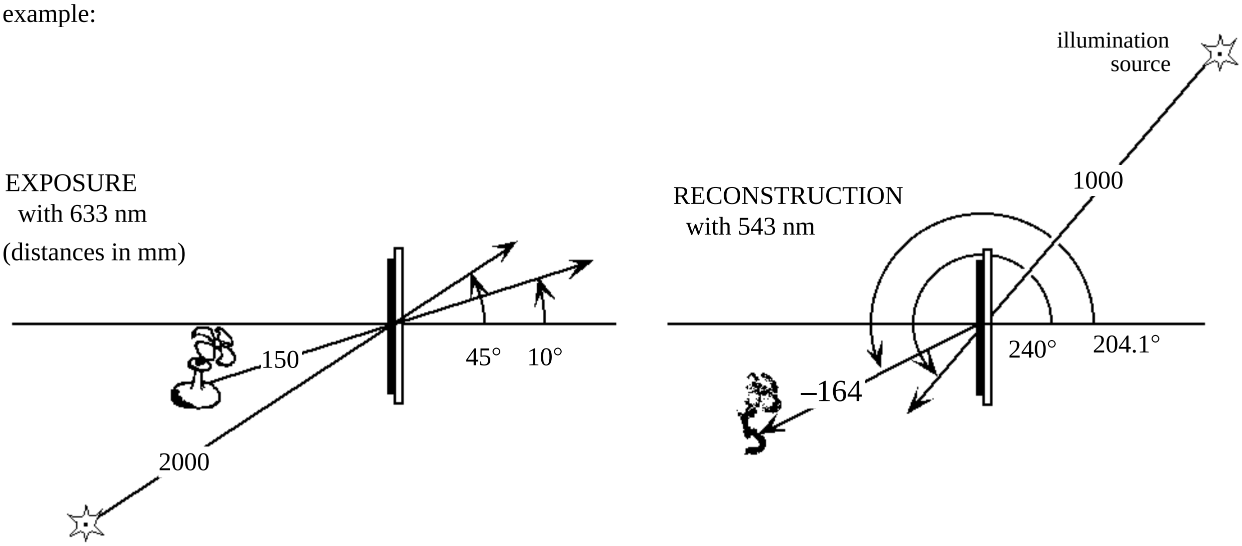



$$\frac{\sin\theta_{out,m} - \sin\theta_{ill}}{\lambda_2} = m\,\frac{\sin\theta_{obj} - \sin\theta_{ref}}{\lambda_1}, \qquad m = 0, \pm1, \pm2, \ldots$$

$$\frac{1}{\lambda_2}\left(\frac{\cos^2\theta_{out,m}}{R_{out,m,V}} - \frac{\cos^2\theta_{ill}}{R_{ill}}\right) = m\,\frac{1}{\lambda_1}\left(\frac{\cos^2\theta_{obj}}{R_{obj}} - \frac{\cos^2\theta_{ref}}{R_{ref}}\right)$$

Magnification - same equations as "direct, vertical" with $m = -1$.

example:




$MAG_{lat}$ = 101%, $MAG_{long}$ = 119%