Physics 23 Laboratory Spring 1987

DIFFRACTION AND FOURIER OPTICS

Introduction

This laboratory is a study of diffraction and an introduction to the concepts of Fourier optics and spatial filtering. The basic principles of diffraction are covered in Chapter 39 of Physics by Ohanian; the principles of Fourier optics and spatial filtering are developed in semi-quantitative form in the theory section below. It is assumed that for many students this material will be new, and that the formal mathematical development will come in later courses. Therefore, we are relying on a descriptive development of the theory and self-discovery through experiment to provide the learning experience. Hopefully, this will provide the background and the stimulus for deeper study in later courses.

Theory

A. Fourier Series

The mathematical theorem of J. B. Fourier, which applies to any periodic function of one variable, can perhaps be best introduced if one thinks in terms of a voltage V(t) as a function of time. The function will have a period τ, the time interval over which it is repeated indefinitely, as illustrated in Fig.(1) below. Fourier showed that V(t) is mathematically (and physically) identical to the sum of a series of sinusoidal voltages of periods τ, $\tau/2$, $\tau/3$, etc. The Fourier components, as these voltages are called, can be sine or cosine functions whose phase and amplitude is determined according to a precise set of formulae. For example, as illustrated in Fig.(1), a square-wave voltage can be synthesized from a series of voltages of frequencies f, 3f, 5f, ..etc., (where $f = 1/\tau$, $3f = 3/\tau$, and so forth). A plot of the amplitudes of the various Fourier components, plotted as a function of frequency, forms the "Fourier spectrum" of V(t). In the case of a square-wave voltage, the amplitudes decrease linearly, with A_n, the amplitude of the component of frequency nf being proportional to 1/n.

There are thus two alternate ways of representing V(t); one in terms of time, the other in terms of frequencies f and their respective amplitudes, A(f). If one knows V(t), one can use the formulae developed by Fourier to calculate each A_n; if one knows the value of each A_n, one can add the components to obtain V(t). In the case of time-varying voltages, the response of an electrical circuit which is linear to any periodic voltage can be deduced from the response to the individual sinsusoidal frequencies which form the Fourier spectrum. For example, if an amplifier can amplify only low frequencies, one knows that it will not reproduce a square-wave accurately, because a square-wave (ideally) has components of infinitely high frequency.

B. Fourier Integrals and Transforms

Fourier series can be applied rigorously only to functions which are strictly periodic; i.e., they repeat indefinitly. For functions which are aperiodic, such as a voltage pulse, one must use more involved techniques to find the Fourier spectrum. Furthermore, rather than a having a discreet set of frequencies, an aperiodic voltage pulse will have a continuous frequency spectrum. The frequency spectrum A(f) of a voltage pulse V(t) is called the "Fourier transform" of V(t). Again, the methods developed by Fourier allow one to calculate A(f) from V(t), and vice-versa. The mathematical formulae for making these calculations are called Fourier integrals. The results can be visualized by studying the examples of aperiodic functions and their Fourier transforms which are illustrated in Fig.(2) below.

C. Spatial Frequencies

So far, Fourier concepts have been applied only to periodic functions of time. Patterns of light and dark, or high and low light intensity, can also be thought of as periodic or aperiodic functions of a spatial variable. For example, an infinite string of black and white stripes can be described as a (one-dimensional) function I(x) in which the intensity I varies with position x across the pattern. The spatial variable x is now analogous to the variable time and the intensity I is analogous to the voltage V. The "period" of this function is the distance λ between points of equal phase, and is measured in, say, centimeters. Whereas the temporal frequency in cycles-per-second (now called Hertz) is the inverse of the temporal period τ, the spatial frequency is the inverse of the spatial period λ, and is measured in cycles-per-cm.

Optical images are always made of objects that are quite limited in spatial extent, certainly not indefinitely repeating. Therefore, one must use Fourier integrals to calculate the Fourier spectra of any real pattern of light and dark. It might be messy, but it can be done. Be it a series of black and white bars, or a black-and-white photograph of your grandmother, one can, in principle, find the Fourier spectrum. There is one additional complication, which has perhaps occurred to you: pictures are two-dimensional, whereas time in only one-dimentional. Thus the intensity distribution of your grandmother's picture is a function of two variables, I(x,y). Such a distribution will have a spectrum of frequencies in two directions, x and y.

A set of horizontal bars (intensity varying in the y-direction) has a corresponding set of "vertical" frequencies, plotted along a vertical (y) axis; a set of vertical bars (intensity varying in the x-direction) has a corresponding set of "horizontal" frequencies, plotted along the horizontal (x) axis; a rectangular grid has both x and y frequency components, resulting in a rectangular array of frequency points.

The Fourier transform of a two-dimensional intensity pattern can itself be thought of as a two-dimensional intensity pattern. The "transform pattern" will be light where a frequency component exists, in proportion to the amplitude of that frequency component, and totally dark where there is no frequency component. The central area of the pattern is the "zero frequency" area, and represents the average (d.c.) intensity level. Points far from the center of the transform pattern represent high frequencies, or rapid changes from light to dark.

The Fourier transform of your grandmother's picture will be more complicated than just a set of discreet x and y spatial frequencies, of course; it will consist of a continuous spectrum of frequencies in both directions. It will be, in a sense, a picture; but it won't look like your grandmother any more than the frequency spectrum of a piccolo note sounds like a piccolo.

D. Fresnel and Fraunhofer Diffraction

The full mathematical analysis of diffraction effects involves some complex techniques, well beyond the level of this course. These techniques are needed if one is to obtain equations for the intensity of the light diffracted in the near vicinity of the diffracting object. The ripples of water waves diffracted by an opening in a barrier in a ripple tank provide an example of what is called "Fresnel diffraction". When the distance of the plane in which the diffraction pattern is observed is made essentially infinte compared with the dimensions of the diffracting object, the analysis is greatly simplified. This latter case is called "Fraunhofer diffraction", and that is what is treated in introductory texts. For example, in Section 39.2 of Ohanian, the rays of light propagating from the slit, and later combining in the diffraction pattern, are parallel; they could only meet at infinity, and the approximation of making them parallel is essential to the argument.

Observing the diffraction pattern at great distances is impractical; for one thing it would be very large, and for another it would be very faint. An alternate, and equivalent, method is to place a lens in front of the diffracting object. Any set of parallel rays from the object striking the lens will then be focussed by the lens in its focal plane, and the intensity pattern formed by all rays will comprise the Fraunhofer diffraction pattern, reduced in size but to scale.

This laboratory will provides an opportunity to compare the two types of diffraction, Fresnel and Fraunhofer. For the former, one places an object in the columnated beam from a laser and observes the "shadow" on a screen about a meter away. For the latter one places a lens one focal length from the object, and observes the pattern at one focal length distance on the othe side. (This relative placement of the lens and the object is not the only one that produces a Fraunhofer-type pattern, but it does make a diffraction pattern that is of the same scale as the object; that is convenient for the procedure described next.)

If a lens placed in front of an object (light intensity pattern) forms a diffraction pattern of that object in the focal plane of the lens, what happens if a second lens is placed in front of the diffraction pattern? It will form a diffraction pattern of the diffraction pattern, which turns out to be just an image of the original object! That is a most significant property of imaging and diffraction that will be explored experimentally in this lab.

E. Diffraction Patterns as Fourier Transforms

The equation for the magnitude of the electric field in a diffraction pattern (viz., Eq. (12), page 887 of Ohanian), if squared and time-averaged, gives the intensity of the light in a diffraction pattern. (Remember that this is a Fraunhofer pattern, and the presence of a lens in from of the diffracting slit can be presumed). This equation essentially transforms one intensity distribution, that of the original object, into a second intensity distribution, that of the diffraction pattern. The figure on the left below [Fig. (3a)] is a one-dimensional plot of the intensity of the light emerging from a long slit of width a (with $a = 3\lambda$). The figure on the right, Fig. (3b), taken from page 887 of Ohanian, is a one dimensional plot of the intensity in the diffraction pattern. If a lens is used to form the pattern, it is the lens which can be thought of as making the transform.

The connection between Fraunhofer diffraction patterns and Fourier transforms is both mathematical and physical. When a lens forms a diffraction pattern of an intensity distribution, it is making a Fourier transform of that distribution from one of space to one of spatial frequencies. The diffraction pattern is the Fourier transform of the object; the focal plane of the lens can be considered then transform plane. If a diffraction pattern is made a second time, it is like making a second Fourier transform, which brings one back to the original object.

Referring back to the discussion of the voltage V(t) and its Fourier spectrum A(f), what the lenses do in this lab is the optical equivalent of changing from the time description of the voltage V(t) to the frequency description A(f), and back again. Furthermore, if one could first transform V(t) to A(f), then modify A(f) in some desired way, and transform back again, one might have an "improved" V(t). For example, suppose V(t) was music plus an annoying 60 Hz "buzz". If one were to filter out electronically just the 60 Hz component of V(t), one would then be left with just the music.

The analogous operation in optics is known as spatial filtering, or optical filtering. The process is one of altering the diffraction pattern of an object, then forming a second pattern which is a modified image of the object. The patterns are altered by placing a mask in them which selectively transmits only certain parts.

The purpose of this lab is to learn by experiment, with many mathematical details being eshewed You are provided with a number of objects, most of the 35 mm slides, with which to form diffraction patterns and double-diffraction images. A quantitative understanding of the relationships between Fourier analysis, Fourier transforms, and optical diffraction should be easier to arrive at after the insight provided by these exercises.

Apparatus

The apparatus in this experiments consists of a board with two parallel tracks of optical bench two meters long; a 0.8 mw He-Ne laser, with a spatial filter; two 100 cm focal length lenses and on 50 cm focal length lens; three lens holders; two adjustable front-surface mirrors; two spring-clip slide mounts; carriers for the above optical components, which are pin mounted, including one carrier with vertical and horizontal adjustments; a set of slides for creating and masking diffraction patterns; a meter stick, a ruler, a flashlight, a magnifying glass, and other small pieces including pins and needles to use as diffracting objects.

Please handle all of this apparatus with care, as it is of good quality and will be degraded or damaged if treated roughly or carelessly. Try to keep finger prints off the lenses, mirrors, and slides, and be careful when you place the carriers on the benches or move them about.

Procedure

A. Initial Adjustments

1. Turn on the laser, and check to see that the spatial filter is producing a clean beam. If this is not the case, ask the instructor to help you get the spatial filter aligned.

2. Adjust the laser mount, and the laser, so that the beam is projected staight down the first optical bench, parallel to the bench in both the vertical and horizontal directions. Keeping the beam at the same height at all points, as measured with a plastic ruler is adequate. Note that the beam expands as it moves down the bench

3. Place the 50cm lens about 50 centimeters from the pinhole, and place a mirror several centimeters further along on the bench. Adjust the mirror so that the beam is reflected back through the 50cm lens, forming a small spot adjacent to the pinhole. Now move the lens to fine tune the focus to make that spot a point. The 50cm lens will now be one focal length away from the pinhole, and the beam should be columnated at about 1 inch diameter at all points on the bench.

B. Fresnel Diffraction Patterns

4. For this exercise the diffracting objects should be placed about 20 - 30 cm beyond the 50 cm lens. The following are suggested: A needle and/or pin, stuck into thecork-and-dowel mount; a razor blade, also stuck in the cork, with the edge half way in the beam; slides #1 (eight holes), #2 (square aperture), and #3 (round hole); the iris diaphram.

   Place each object, in turn, as suggested, and observe the diffraction pattern at several distances from it. Distances of 10, 30, 100, and 300 cm are suggested. For the longer distances it will probably be necessary to "bend" the beam around a corner, and back down the second bench. Hold a piece of white paper in the beam to observe the pattern, or place a screen on the bench, or place one of the translucent filters in a slide holder and observe it from the back side.

Note that the diffraction pattern changes from a fairly sharp shadow, to a fuzzy shadow, to a pattern that is not really shadow-like at all. Note also how the pattern of the iris diaphram changes with aperture diameter. The change in the patterns is the transition from the Fresnel, or near-field pattern, to the Fraunhofer, or far-field pattern.

Sketch the patterns in your notebook.

C. Fraunhofer Diffraction Patterns

5. The full transition to a Fraunhofer-type pattern can be achieved by introducing a second lens, with a 100 cm focal length. The second lens should be placed one focal length from the diffracting object, forming a diffraction pattern one focal length to the other side and a real image at infinity. Place the lens 1 meter from the object, and place a mirror a short distance behind it. Adjust the mirror so that the beam, reflected back through the lens, forms an image on the object slide holder. Now move the 100 cm lens until the image is sharp; this places it one focal length from the object.

The diffraction pattern of any object placed in the slide holder will be formed 100 cm to the right of the second lens (this will actually be a point on the second bench where the beam will have been deflected by the two mirrors).

6. Observe the Fraunhofer diffraction patterns of some of the following slides:
   #4, #5, and #6 - parallel lines of various spacing - when mounted vertically, horizontally, or at an angle;
   #7, #8, and #9 - concentric circles of various spacings;
   #10, #11, and #12 - grids of various fineness.

   Compare the patterns for the grids with that of the lines; note how the pattern changes when the structure of the object becomes finer.

   Other sides and objects can be used, such as the variable aperture. Sketch the patterns in your notebook, paying attention to the spacing of the maxima (bright lines and dots) in the pattern, which is a measure of the spatial frequencies in the object.

D. Double Diffraction

7. Now place a second 100cm lens 1 meter beyond the diffraction pattern. It will form 1 meter away on the other side a diffraction pattern of the diffraction pattern; which, as you will see, is an image of the original object.

   The exercises suggested for this part are simply ones of placing various slides in the object holder, as was done previously, and now observing the results of double diffraction.

E. Spatial Frequencies and Spatial Filtering

8. The images formed in the previous exercise were not perfect, because the full diffraction pattern could not be transmitted by the finite apertures of the lenses. Further degradation or modification of the images can be introduced by placing a mask beside the diffraction pattern, a point half-way (optically) between the two 100cm lenses.

   Four masks are suggested for these exercises: slide #15, a slit in a black screen; slide #25, a slit with an opaque center; slide #26 (homemade), a glass slide with a black bar which has a central gap; and the iris diaphram.

   As an indication of what can be learned in this section, review or re-examine the diffraction patterns of slides #5 and #11, noting how the pattern of #5 changes with orientation. Put #11 in the object-slide holder, and the mask #15 (or the iris diaphram) in the mask holder. Set the mask holder in the adjustable mount, and place it in the diffraction plane, where the diffraction pattern is located.

9. Observe the effects of masking off various parts of the first diffraction pattern on the second diffraction pattern (i.e., the image). Note that by modifying the pattern of slide #11 to replicate the pattern of slide #5, that one can transform the image of slide #11 to look (approximately) like the image of slide #5.

As has been discussed, the diffraction pattern is essentially a transform of the intensity-vs-position of the object into amplitude-vs-spatial frequency. The zero-frequency, or d.c., amplitude is the central spot of the pattern; higher spatial frequencies and finer detail are carried by points away from the center. Thus, one would expect that by cancelling out the outer part of the pattern and retaining the center that gross deatil would be retained but fine detail would be lost.

10. Try masking off the diffraction pattern of various objects to include more or less of the outer points, or higher spatial freqencies, and observe how this affects the detail in the image.

F. Image Processing

This exercise shows how one can enhance images and bring out particular details by modifying a diffraction pattern with optical filtering. Slide #22 is a simulation of a photograph of cloud chamber tracks. The horizontal lines represent tracks from incoming particles, the more random tracks represent the particles that are products of interactions. It is the latter tracks that one wants to enhance, the former tracks one wants to surpress.

11. Mount slide #22 in the object holder, and observe the diffraction pattern and the image. Note that component of the pattern - a line of closely-spaced maxima - that corresponds to the lines. Now use the glass slide with the black bar to filter out that line. Describe the results in your notebook.

12. Mount slide #24, a half-tone picture of a physicist, in the object holder, and observe the diffraction pattern and the image. Note the resemblence of this diffraction pattern to that of slide #11, and the grainy nature of the image. Now use the iris diaphram to filter out the high spatial frequencies of the pattern, and so convert the grainy half-tone picture to a smoother continuous one. Describe the results in your notebook.

13. Finally, mount slide #25, a multiple-image photo of two people, in the object holder. One image is stored with vertical shaded lines, the other with horizontal shaded lines; the diffraction pattern resembles that of a grid. Use the mask slide #23 to select out the pattern of one or the other image, observe the result, and describe them in your notebook.

G. Babinet's Principle

14. Read Section 39.3 of Ohanian. Two slides have been made of reduced copies of Fig.39.17 on page 894. Below that figure is a picture of the diffraction patterns formed by the complementary pair of object masks. Compare the diffraction patterns that you observe from these slides with the ones given in the text.