Department of Electrical and Computer Engineering
University of Colorado at Boulder

OPTICS LAB -ECEN 5606

Kelvin Wagner

Experiment No. 10

FOURIER OPTICS

1 Introduction

Fourier optics is one of the most powerful tools available for the spatial frequency analysis of images, and for the construction of real time correlators. One property that makes optics an exciting tool for frequency analysis is that, in free-space propogation of light, one can see the effects of lenses and filters on a light beam as the beam propogates through the system.

In this experiment you will set up a system of lenses that allow the display and modification of the Fourier spectrum of two-dimensional objects. One can then use a variety of filters from simple opaque objects to computer generated holograms that will selectively block certain spatial frequencies of the original object. These devices can be used either to modify the characteristics of the object or to perform various computations to extract information out of the object. By using complex transparencies in the Fourier plane, implemented holographically, you will be able to build matched spatial filter correlators for object recognition and location.

2 Background

Any light wave of finite spatial extent undergoes diffraction as it propogates through space. If at a certain position z_1, there is a transparency with an amplitude transmitance described by $u_1(x, y)$, at a position z_2 far from z_1 ($z = z_2 - z_1$), the amplitude variation is given in the Fraunhofer approximation by

$$u_2(x_2, y_2) = \frac{e^{ikd}}{i\lambda z} e^{i\frac{2\pi}{\lambda z}(x_2^2+y_2^2)} \mathcal{U}_1(\frac{x_2}{\lambda z}, \frac{y_2}{\lambda z})$$

where $\mathcal{U}_1$ is the angular spectrum (Fourier transform) of u_1 This approximation is only valid for distances far from z_1. When a lens is inserted one focal length from the transparency, it introduces a quadradic phase factor ($e^{-i\frac{2\pi}{\lambda F}(x^2+y^2)}$) onto the wave that produces at the back focal plane of the lens an exact Fourier transform of the transparency (i.e. no Fraunhofer approximation is needed and the quadratic phase factor can be elliminated). This result is the essence of Fourier optics and is what makes it usefull for a variety of applications.

If the object's amplitude transmittance is $a(x, y)$, then at the back focal plane of an ideal spherical lens we will observe the field distribution

$$A(x', y') = \frac{1}{i\lambda F} \int\int a(x, y) e^{-i\frac{2\pi}{\lambda F}(xx'+yy')} dxdy.$$

In this expression x' and y' are the physical coordinates in the output plane, and we can define a normalized set of output coordinates $u = x'/\lambda F$ and $v = y'/\lambda F$ which are spatial frequency variables measured in cycles/mm. Thus $A(x', y')$ is the Fourier transform of $a(x, y)$ scaled by the factor λF. What we actually observe with the eye is the intensity in the Fourier plane given by $I = |A|^2$, so we are really viewing the power spectrum of the input object. When we need the full complex Fourier spectrum, as in the matched spatial filter, then we can use holographic techniques.

3 Preparation

Read the sections on Fourier optics in your favorite coherent optics book.

- Saleh and Teich, Photonics, Ch.4
- J.W. Goodman, Introduction to Fourier Optics, McGraw Hill (1968), 5.2,7.4-6.
- Hecht and Zajac, Optics, Addison-Wesley 1976, chapter 11.
- J. D. Gaskill, Linear Systems, Fourier Transforms, and Optics, John Wiley (1978).
- W.T. Cathey, Fourier Optics and Holography.

4 Prelab

1. A wire screen with .1mm cells, and wire thickness of 25μm is illuminated by a .6328μm laser and the diffracted light is Fourier transformed with a lens with a focal length of F=250mm. Remember, when the grating is illuminated the wires are opaque and the space in between passes light. Sketch and dimension the Fourier plane. What size slit should be used to remove all but the first order diffraction in the x direction, and all orders in the y direction. Which diffraction maxima is missing for this particular ratio of wire thickness to wire spacing?

2. Sketch the Fourier transform of the letters A,E,W,F and O. Are all these letters distinguishable by their Fourier spectra?

3. Sketch the autocorrelation of a circle $\circ$ with itself, and a disk $\bullet$ with itself, and of the crosscorrelation of the circle with the disk, what does this illustrate about edge enhanced matched spatial filters?

4. [NOT REQUIRED] Correlators in general, and optical matched spatial filters in particular, are quite sensitive to the change in scale of an object with respect to the scale of the reference with which the Van der lugt filter was recorded. An interesting approach to overcoming this problem is to place the input object transparency on a translation stage in the converging beam of a Fourier lens that is illuminating the matched filter, and translate the transparency forwards and backwards while examining the output plane and searching for the best correlation peak, which corresponds to the scale compensated correlation. Analyze this system, and show that it indeed performs a scale compensating correlation, at some position along the optical axis for the input transparency, while at other positions a scale mismatched crosscorrelation is produced.

5 Set Up

This section has been added in order to help the person setting up the lab experiment and the student who will be doing the lab. This lab set up is intended to be done once and then left alone in order to reduce the amount of work done by the students.

Using the diagram in Figure 1 as a guide, align the HeNe laser with the table as usual using the multiple iris method.

Spatial filter the beam and collimate the expanding output with a lens that produces a beam at least 40 mm in diameter (leave room before the spatial filter for ND filters, the shutter and the mirror used in part one of the procedure). Set up the two lens telescopic imaging system shown in the bottom of Figure 1. Leave room for the removable mirrors and the object. The object should be placed one focal length before the first FT lens, and the image should appear one focal length beyond the second FT lens, and the separation between the two lenses should be the sum of their focal lengths. Have your TA show you some of the tricks that can be used to ensure this alignment, (auto-collimation condition, speckle size maximization). Make sure that the imaging system produces a sharp image of an object placed in the input plane. Align the mirror and the camera so that the Fourier transform plane is imaged on the CCD array. Make sure the spot on the CCD is as sharp as possible.

Set up the path length matched holographic interferometer that uses the dynamic photoanisotropic optical media (DPOM) as a Vander Lugt filter as shown in the upper part of Figure 2. Using a Fourier transform lens a distance F beyond the DPOM film plate, focus the reference beam to a spot on the output observation plane. Make sure that the DPOM film plate is perpendicular to the object beam, to minimize depth of field requirements on the Fourier plane, and that the Fourier transform lens is exactly one focal length before the film. The object should be placed one focal length before the FT lens on a stage with both translation and rotation.

5.1 Materials and Equipment

- Doubled YAG or Argon laser
- HeNe laser
- Dynamic Photoanisotropic Optical Material
- CCD Camera
- ND Filters
- Polarizer
- half wave plate
- 2 Aperture stops
- 6 Positive (100 - 250 mm) lenses
- 5 Mirrors
- Beam (non-polarizing) splitter
- Kinematic mount
- High resolution IC mask
- translation and rotation stage mount

6 Procedure

1. **Single Slit Diffraction**

   Temporarily place the variable slit in the beam, close it down all the way and describe the pattern observed on a card 10cm behind the slit as you slowly open it. Do you see a sinc^2 pattern? Curl a piece of paper into a 10cm diameter half cylinder, and place the axis of the cylinder at the slit position, what do you see on the paper, and what is the difference between this and the pattern seen on the flat card. Why do you see this pattern if we are not in the Fourier plane of a lens? For a particular slit width or a particular distance from the slit, where does this far field approximation become invalid?

2. **Fourier Transformation**

   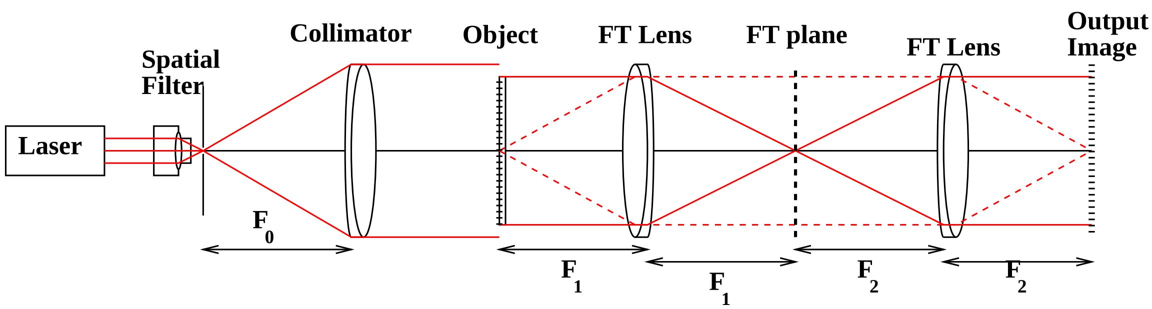

   Figure 1: 4F optical system for spatial filtering

   Spatial filter the beam and collimate the expanding output with a lens that produces a beam at least 40mm in diameter (leave room before the spatial filter for ND filters and a shutter). Set up the two lens telescopic imaging system shown in Figure /reffig:4F The object should be placed one focal length before the first FT lens, and the image should appear one focal length beyond the second FT lens, and the seperation between the two lenses should be the sum of their focal lengths and should be checked using the collimation tester. Have your TA show you some of the tricks that can be used to ensure this alignment, (collimation tester, autocollimation condition, speckle size maximization) and describe the procedure that you used. Make sure that the imaging system produces a sharp image of an object placed in the input plane. Place the wire mesh, object A, in the input plane, and describe what you see in the Fourier plane, you may wish to look at a magnified image of the Fourier plane on the far wall. What is the wire thickness/spacing ratio for this mesh? Translate the object and describe the effect on the Fourier plane. Is it translationally invariant (at least as far as the eye can see)? Rotate the input transparency, and describe the effect on the Fourier plane. Is the Fourier plane rotationally invariant? Place the object after the first Fourier transform lens and move it along the optical axis, and describe the effect on the Fourier plane.

3. **Spatial Filtering of Periodic Objects**

   Place the symmetrically opening variable slit in the Fourier plane, with the slit vertical, and centered on the DC spot. Describe your procedure for making sure the slit is actually in the Fourier plane, and not in front of or behind it, and describe how you centered the slit in the Fourier plane. Describe what happens to the image as you open the slit slowly. Rotate the slit by 90 degrees and perform the same operation, noting your observations, and sketching the output.

4. **Spatial Filtering**

   Insert one of the block letter objects into the input plane of the Fourier spatial filtering system. Carefully align a microscope slide with a small opaque dot in the center of the Fourier plane, thereby blocking out all of the low spatial frequency components. How critical is the transverse and longitudinal alignment of this high pass spatial filter, and how critical would these alignments be for a 25μ opaque DC block? Observe and describe the output plane. What processing operation has been performed on the input object, and why would this be useful?

5. **Computer Generated Holograms**

   Image the Fourier transform with a high magnification on the far wall. Replace the input object with one of the CGH masks from the B series and observe the Fourier plane and the magnified image and describe what you observe. Replace with one of the CGH masks from the C series and describe what you observe in the Fourier plane. Compare the two types of CGHs under the magnification of the loop and describe the differences in the masks. They are both Lohmann style Fourier transform holograms with one important difference, can you tell what the difference is in the encoding algorithm?

6. **Matched Spatial Filtering and the Van der Lugt Correlator**

   Set up the path length matched holographic interferometer that uses the dynamic photoanisotropic optical material (DPOM) as a Van der Lugt filter as shown in Figure 2. Choose a high resolution object such as an IC mask mounted on a translation and rotation stage. Using a Fourier transform lens a distance F beyond the DPOM, focus the reference beam to a spot on the output observation plane. The DPOM plate should be perpendicular to the object beam, to minimize depth of field requirements on the Fourier plane, and the Fourier transform lens should be exactly one focal lenth before the film. The object should be placed one focal length before the FT lens. Measure the spatial distribution of power in the DPOM film plane, and adjust the reference beam power to be uniform and essentially equal to the information bearing wings of the object Fourier transform as illustrated in Figure 3. When recording, do not worry about saturating the DC spot, you actually would like to avoid recording any fringes in the low spatial frequency region of the hologram in order to form a high pass spatial filter (or edge enhanced hologram), because such a hologram has very good recognition and descrimination capabilities as compared with an all pass spatial filter (prelab problem 3). Record a hologram for a few seconds. Now block the reference beam, and reilluminate the DPOM with the Fourier transform of the object. Observe the output screen and locate the correlation peak. You will find that the correlation peak decays as the hologram is read out due to erasure of the DPOM. Try recording the hologram for different amounts of time, and measure the erasure time as a function of the recording time. Use the CCD camera to observe the fine structure of the correlation peak. Translate the input object, and observe the correlation peak. Is the correlator space invariant? Rotate the input object and observe the correlation peak. Is the correlation rotation invariant? What is the rotational

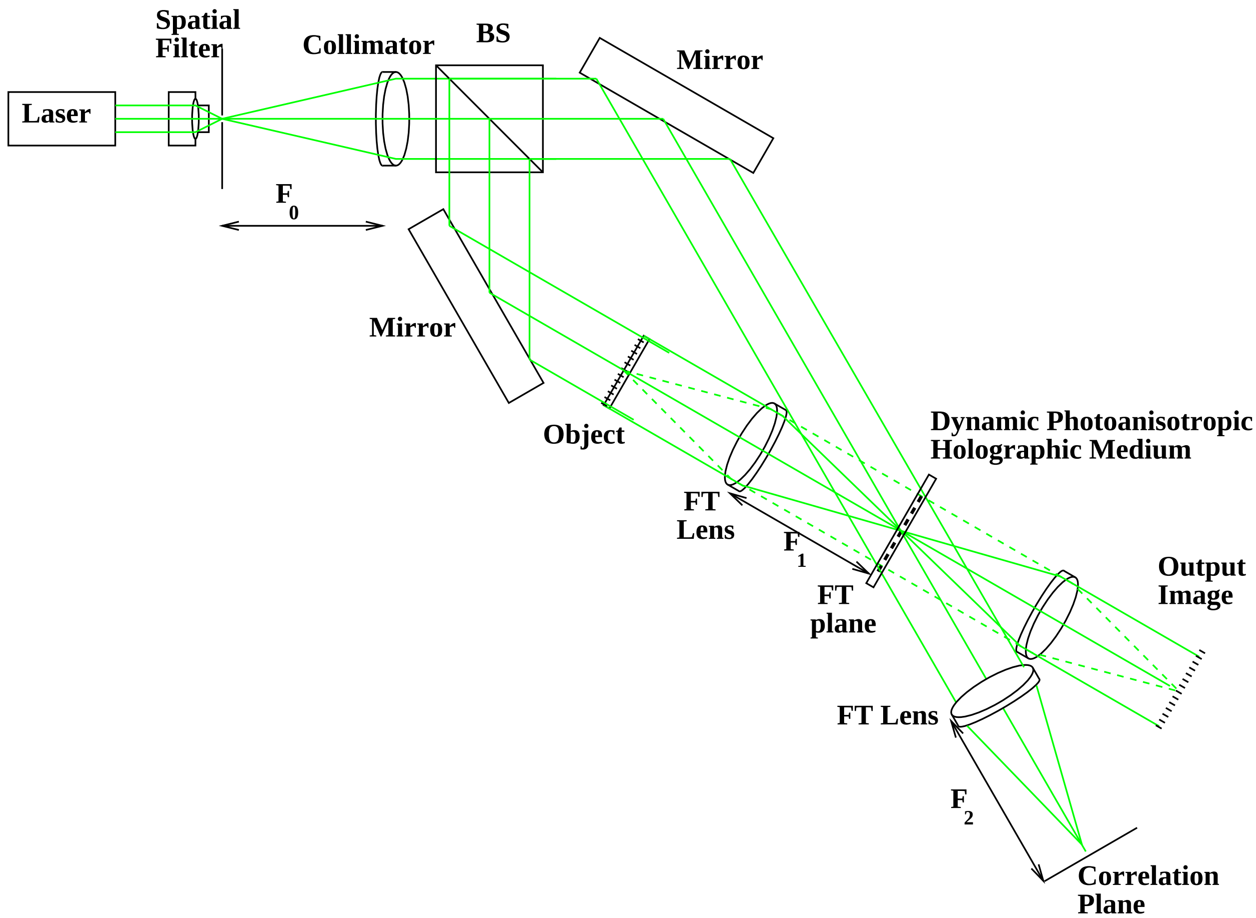

Figure 2: Van der Lugt holographic matched spatial filter system using dynamic photoanisotropic media.

sensitivity of the correlator? Block off most of the input object and use just a small piece of the object as the input. What is the effect on the correlation peak?

Try illuminating with the reference beam, to produce a reconstruction of the edge enhanced object which is stored in the hologram as the matched spatial filter template. If sufficient diffraction efficiency is available to see the reconstruction, use the CCD to take a picture of your edge enhanced reconstruction and the original input object.

7. **Polarization holography**

   Put a half wave plate in the reference beam, to rotate the polarization by 90 degrees with respect to the object beam. Now try to record a hologram with these orthogonally polarized beams. Do you see any diffraction from the DPOM and if so what is the polarization of the diffracted light? Can you record holograms with orthogonally circularly polarized light? Which holograms have the highest diffraction efficiency? Do the polarization holograms decay at the same rate as the intensity holograms?