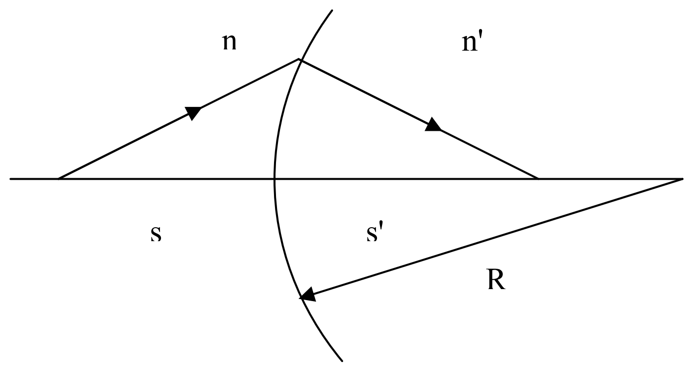
PHYC 3540
Summary

I. Propagation of light

a. History, wave vs. particle picture
b. Fermats Principle – actual path is that for which the OPL is an extremum
c. Huygen's principle – every point on wavefront acts as a source
   i. Large aperture (>>λ) – rectilinear propagation (Geometric optics)
   ii. Small aperture (≥ λ) – spreading of wavefront (Diffraction)
d. Wavefronts, Rays – Laws of reflection and refraction (Snels law)

II. Geometric Optics

a. Imaging – Cartesian surfaces, Paraxial ray approximation
b. Refraction at a Spherical interface



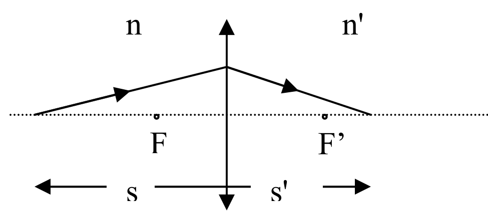
$$\frac{n}{s}+\frac{n'}{s'}=\frac{n'-n}{R}=\frac{n}{f}=\frac{n'}{f'}$$

$$\frac{n}{f}=\frac{n'}{f'}=P\equiv Power$$

c. Thin lens

$$\frac{n}{s}+\frac{n'}{s'}=\frac{n_L-n}{R_1}-\frac{n_L-n'}{R_2}=\frac{n}{f}=\frac{n'}{f'}=P$$

$$Magnification \quad M=\frac{h'}{h}=-\frac{ns'}{n's} \quad for \quad all \quad imaging$$

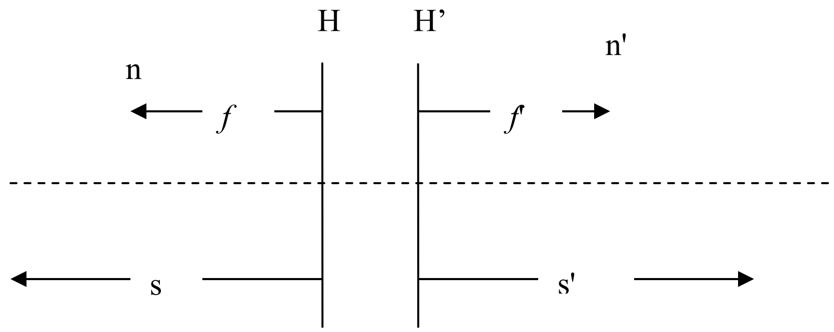


d. Principal Planes (H,H')

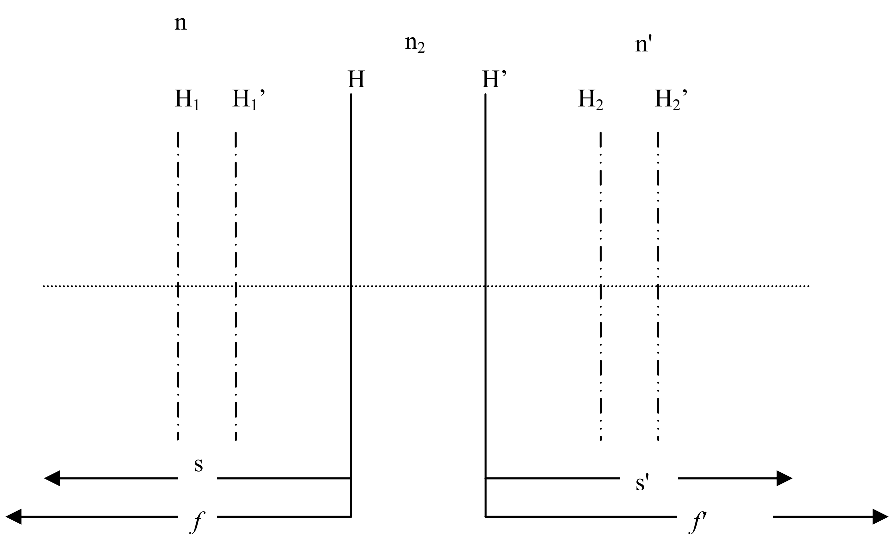
   i. Used to represent any optical element or system and are located such that the system images an object according to:

$$\frac{n}{s}+\frac{n'}{s'}=\frac{n}{f}=\frac{n'}{f'}=P$$

   ii. The principal planes used to represent a spherical interface (or thin lens) must coincide at the interface (or centre of the thin lens).



e. Combination of two systems



   i. Power of combined system $P=P_1+P_2-\frac{dP_1P_2}{n_2}$

where, $P_1=\frac{n}{f_1}=\frac{n_2}{f_1'}$ and $P_2=\frac{n_2}{f_2}=\frac{n'}{f_2'}$

ii. The location of H, H' are determined with respect to H_1 and H_2' as follows:

$$h' = H_2'H' = -d\left(\frac{P_1}{P}\right)\left(\frac{n'}{n_2}\right) = -d\,\frac{f'}{f_1'}$$

$$h = H_1H = d\left(\frac{P_2}{P}\right)\left(\frac{n}{n_2}\right) = d\,\frac{f}{f_2}$$

f. Matrix methods in paraxial optics

i. Matrices

Translation matrix $\begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$

Refraction matrix $\begin{bmatrix} 1 & 0 \\ \dfrac{n-n'}{Rn'} & \dfrac{n}{n'} \end{bmatrix}$

Reflection matrix $\begin{bmatrix} 1 & 0 \\ \dfrac{2}{R} & 1 \end{bmatrix}$

Thin lens matrix $\begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix}$

ii. Cardinal point locations

$$h' = -d\,\frac{P_1}{P}\left(\frac{n'}{n_2}\right) = \frac{1-A}{C}$$

$$h = d\,\frac{P_2}{P}\left(\frac{n}{n_2}\right) = \frac{D-\left(n/n'\right)}{C}$$

g. Stops in optical systems

i. Aperture stop

- Element that limits the bundle of rays (and hence light gathering power or speed) collected by optical system
- Is that element whose entrance pupil subtends the smallest angle at the object point in question
- Exit pupil is the image of the aperture stop in all elements following it
- Entrance pupil is the image of the aperture stop in all elements preceding it

ii. Field stop
- Element of the optical system that limits the field of view
- That element whose entrance window subtends the smallest angle at the centre of the entrance pupil
- Exit window is the image of the field stop in all elements following it.
- Entrance window is the image of the field stop in all elements preceding it

iii. Chief ray, marginal rays – definition of edge of field of view

iv. Angular field of view
- Image space – angle formed by the entrance window as viewed from the centre of the entrance pupil
- Object space – angle formed by the exit window as viewed from the centre of the exit pupil

h. Applications

i. Cameras (aperture settings (f#), exposure, telephoto lens)

ii. Human eye – myopia, hyperopia

iii. Hand magnifiers, eyepieces, compound microscope

iv. Telescope, binoculars

III. **Physical optics** (wave properties are important)

a. <u>Wave equation</u> – plane waves, spherical waves, cylindrical waves

b. <u>Amplitude reflection and transmission</u> co-efficients

$$\rho_{12} = \frac{v_2 - v_1}{v_1 + v_2} \quad ; \quad \tau_{12} = \frac{2v_2}{v_1 + v_2}$$

c. <u>Electromagnetic waves</u> (E ⊥ B; E, B in phase, speed c/n)

i. Irradiance

$$I = \frac{1}{2}\varepsilon v E_o^{\ 2} \quad ; \quad E_o = Amplitude$$

ii. Transmittance

$$T = \frac{I_2}{I_1} = 4\frac{n_1 n_2}{(n_1 + n_2)^2} = \tau_{12}\tau_{21} = \frac{n_2}{n_1}\tau_{12}^{\ 2}$$

iii. Reflectance (1→2)

$$R = \frac{I_R}{I_1} = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 = \rho_{12}^{\ 2} = \rho_{21}^2$$

d. <u>Interference</u>

i. Two source interference
- Irradiance

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi_2 - \phi_1)$$

$$where, \quad \phi_2 - \phi_1 = \delta = k(r_2 - r_1) + (\varepsilon_2 - \varepsilon_1)$$

- If $\varepsilon_1 = \varepsilon_2$ and $I_1 = I_2 = I_o$ there are maxima located at,

$$\sin\theta = \pm m \frac{\lambda}{a}$$

- Examples
  - Double slit, Lloyds mirror, radio towers
  - Dielectric layers, e.g. Haidinger bands, fringes of equal thickness observed in oil films, air wedges
  - Antireflection coatings
  - Newton's rings

ii. Multiple beam interference (coated dielectric layers)

- Transmitted irradiance

$$I_T = \frac{T^2 I_o}{(1-R)^2}\left(\frac{1}{1+\frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}\right)$$

where, $\frac{\delta}{2} = k_o nd\cos\theta'$ and n, θ' pertain to the dielectric layer.

  - Irradiance maxima occur when

$$2nd\cos\theta' = \pm m\lambda_o$$

- Fabry-Perot Interferometer
  - Finesse =

$$\Im = \frac{separation \quad between \quad fringes}{fringe \quad width \quad (FWHM)} = \frac{\pi\sqrt{R}}{1-R}$$

  - Free spectral range

$$\Delta\lambda_{FSR} = \frac{\lambda^2}{2nd} \quad ; \quad \Delta\tilde{\nu}_{FSR} = \frac{1}{2nd}$$

  - Fringe width

$$\Delta\lambda = \frac{free \quad spectral \quad range}{finesse} = \frac{\lambda^2}{2nd\Im}$$

  - Spectral Resolution

$$\Re = \frac{\lambda}{\Delta\lambda_{\min}} = \frac{2nd\Im}{\lambda}$$

$$\Re = \frac{\tilde{\nu}}{\Delta\tilde{\nu}} = 2nd\tilde{\nu}\Im$$

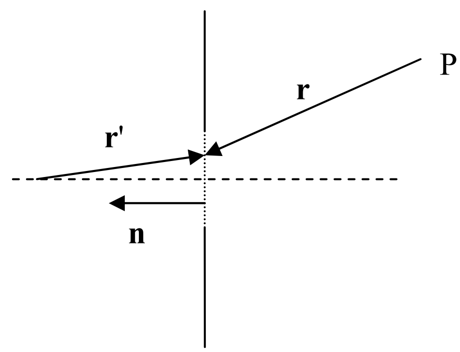
e. Diffraction

i. Fresnel–Kirchoff diffraction formula (Mathematical statement of Huygen's principle)

$$E = \frac{-ikE_o}{2\pi}\int_{aperture}\frac{e^{ik(r+r')}}{rr'}F(\theta)dA$$

where,

$$F(\theta) = \frac{1}{2}\left[\cos(\hat{n},\vec{r}) - \cos(\hat{n},\vec{r}\,')\right] = obliquity \quad factor$$



ii. Fraunhofer limit (plane waves)

diffraction integral reduces to $E = C'\int_{aperture} e^{ikr}dA$

- Single slit (width b)

$$I = I_o\left(\frac{\sin\beta}{\beta}\right)^2 \quad \text{where, } \beta = \frac{kb\sin\theta}{2}$$

$$I = 0 \ \text{ when } \sin\theta = \pm m\frac{\lambda}{b}$$

- Circular aperture (diameter $d_o = 2a_o$)

$$I = I_o\left(\frac{2J_1(\gamma)}{\gamma}\right)^2 \quad \text{where, } \gamma = ka_o\sin\theta$$

$$I = 0 \quad \text{when } \sin\theta = \frac{1.22\lambda}{a_o}$$

Airy disc; Spatial resolution of optical devices; focussing limit

- N slits (width b, separation a)

$$I = I_o\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\alpha}\right)^2 \quad \text{where, } \alpha = \frac{ka\sin\theta}{2}$$

Principal maxima when $\sin\theta = \pm p\frac{\lambda}{a}$

- Diffraction gratings
  - Grating dispersion
  - Spectral resolution $\Re = mN$

f. Polarization

i. Jones vector for polarization

General $\begin{bmatrix} E_{ox} \\ E_{oy}e^{i\delta} \end{bmatrix}$

Vertical $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ Horizontal $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Linearly polarized at an angle ±α, $(-1)^m \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$

Left Circularly polarized $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$(i.e. counterclockwise)

Right Circularly polarized $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$ (i.e. clockwise)

Left Elliptically polarized $\frac{1}{\sqrt{A^2 + B^2 + C^2}}\begin{bmatrix} A \\ B + iC \end{bmatrix}$

Right Elliptically polarized $\frac{1}{\sqrt{A^2 + B^2 + C^2}}\begin{bmatrix} A \\ B - iC \end{bmatrix}$

where, $E_{ox} = A \quad E_{oy} = \sqrt{B^2 + C^2} \quad \delta = \tan^{-1}\left(\frac{C}{B}\right)$

and the angle of inclination, α, of the ellipse is given by

$$\tan(2\alpha) = \frac{2E_{ox}E_{oy}}{E_{ox}{}^2 - E_{oy}^2}$$

ii. Jones matrices for optical elements

- Linear polarizer (Transmission axis at an angle θ wrt x-axis)

$$\begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

- Phase retarders

  In general, $\begin{bmatrix} e^{i\varepsilon_x} & 0 \\ 0 & e^{i\varepsilon_y} \end{bmatrix}$where, ε_x and ε_y represent the advance in phase of the components

  - Quarter wave plate ($|\varepsilon_y - \varepsilon_x| = \pi/2$)

    $e^{-i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ Slow axis vertical

    $e^{+i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$ Slow axis horizontal

- Half wave plate ($|\varepsilon_y - \varepsilon_x| = \pi$)

$$e^{-i\pi/2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{ Slow axis vertical}$$

$$e^{+i\pi/2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{ Slow axis horizontal}$$

- Rotator (light polarized at an angle θ is rotated by an angle β)

$$\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$$

g. <u>Holography</u>

i. Hologram

$$R(x,y,z) = r(x,y,z)e^{i\psi(x,y,z)} = re^{ikz}$$

$$O(x,y,z) = o(x,y,z)e^{i\phi(x,y,z)} = oe^{ikr}$$

$$where \qquad r = \sqrt{x^2 + y^2 + z^2}$$

$$I(x,y) \propto |O+R|^2 = OO^* + RR^* + OR^* + O^*R$$

where, R – reference wave, O – object wave, I - intensity

ii. Reconstructed wave

$$R(x,y)t_p = (t_b + BOO^*)R + BR^2O^* + B|R|^2 O$$

where, t – transmission, first term – direct wave, second term – conjugate wave, third term – object wave