Department of Electrical and Computer Engineering
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OPTICS LAB -ECEN 5606

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Experiment No. 12

CRYSTAL OPTICS

1 Introduction

Crystal optics studies the propagation of electromagnetic waves in anisotropic media and uses these affects as key components for manipulating, controlling, and analyzing the state o polarization. In this lab you will learn about the propagation of light through anisotropic dielectric crystals, and how to use the phenomena of birefringence and dichroism to manipulate the state of polarization of an optical wave. In addition you will learn how to determine the optical axes of an anisotropic crystal by using polarization holography to generate conoscopic interference figures.

2 Background

2.1 Crystal Optics

In isotropic materials, the electric displacement vector $\vec{\mathbf{D}}$ is parallel to the electric field vector $\vec{\mathbf{E}}$, related by $\vec{\mathbf{D}} = \epsilon_0\epsilon_r\vec{\mathbf{E}} = \epsilon_0\vec{\mathbf{E}} + \vec{\mathbf{P}}$, where $\epsilon_0 = 8.854 \times 10^{-12}$F/M in MKS and ϵ_r is the unitless relative dielectric constant, and $\vec{\mathbf{P}}$ is the material polarization vector. In an anisotropic crystal, the vectors $\vec{\mathbf{D}}$ and $\vec{\mathbf{E}}$ are no longer parallel and their relationship is determined by the tensor relation, $\vec{\mathbf{D}} = \epsilon_0\overline{\overline{\epsilon}}_r\vec{\mathbf{E}}$, where $\overline{\overline{\epsilon}}_r$ is the relative dielectric susceptibility tensor, and the dependence of polarization $\vec{\mathbf{P}}$ on $\vec{\mathbf{E}}$ is $\vec{\mathbf{P}} = \epsilon_0\overline{\overline{\chi}}\vec{\mathbf{E}}$, where $\overline{\overline{\chi}}$ is the unitless susceptibility tensor. For nonabsorbing crystals, there always exists a set of orthogonal coordinate axes (note that in low symmetry crystals these principal axis may be a function of temperature or wavelength), called principal dielectric axes, such that both the $\overline{\overline{\epsilon}}$ and the $\overline{\overline{\chi}}$ tensors assumes a diagonal form:

$$\epsilon_{ij} = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \qquad \chi_{ij} = \begin{bmatrix} \chi_{11} & 0 & 0 \\ 0 & \chi_{22} & 0 \\ 0 & 0 & \chi_{33} \end{bmatrix}$$

The principal dielectric constants are $\epsilon_{ii} = 1 + \chi_{ii}$, and the principal indices of refraction experienced by a wave purely polarized along the i principal axes are $n_i = \sqrt{\epsilon_{ii}}$ (but note the refractive index is not a tensor). $\vec{\mathbf{D}}$ and $\vec{\mathbf{P}}$ are not necessarily parallel to $\vec{\mathbf{E}}$, but are parallel for principal axes polarizations. This tensorial relation causes the index of refraction to vary with the direction of propagation and the state of polarization. When, $\chi_{11} = \chi_{22} \neq \chi_{33}$, the crystal is called uniaxial, and has a rotational symmetry of the 2nd rank properties around the $3 = z$ axis. Most of the crystals used in this lab and in most optical experiments are uniaxial. In a uniaxial crystal, the unique element of the susceptibility tensor determines the optic axis, and by convention is always aligned in the $3 = z$ position. If the extraordinary index is less than (greater than) that along the two orthogonal axes, the crystal is termed negative(positive) uniaxial. The crystal structure determines the degree and nature of the anisotropy.

As light enters an anisotropic dielectric medium it is resolved into two orthogonally polarized eigenmodes of polarization, which are characterized by two independent velocities of phase propagation. An extraordinary ray is a beam of light whose electric-field vector is polarized in the same plane as the optic axis, and due to the anisotropy experiences a phase velocity which depends on direction of propagation. An ordinary ray has its electric-field vector polarized normal to the optic axis, and due to this symmetry has a phase velocity independent of direction, just as in an isotropic media. Now suppose unpolarized light (or circularly polarized light) is incident on an anisotropic crystal. The eigencomponents separately refract into and independently propagate through the crystal so that two beams polarized orthogonally to each other exit the crystal. This phenomenon is known as birefringence.

There are numerous important consequences of this effect for the manipulation and analysis of the polarization state of an optical beam. Important effects include birefringence, dichroism, anisotropic refraction, anisotropic reflection, walkoff, and optical activity. These effects can be used to make quarter wave plates, half wave plates, variable wave plates, dichroic polarizer, polarizing beamsplitting prisms, depolarizers, optoisolators, and other polarization manipulation devices.

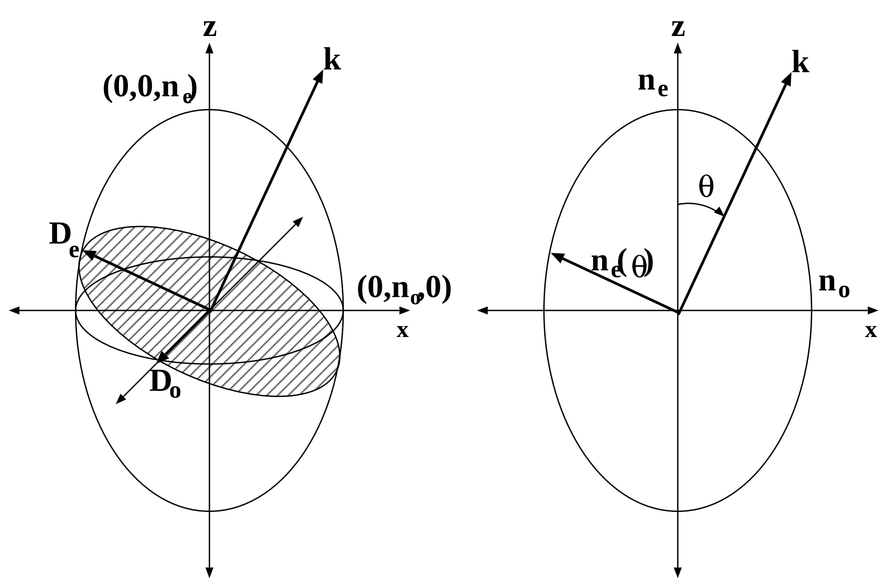


Figure 1: Optical indicatrix for a uniaxial crystal. For each direction of propagation, $\vec{k}$ the orthogonal elliptical plane can be diagonalized to find the eigendirections for the electric displacement vectors, $\vec{\mathbf{D}}_e$ and $\vec{\mathbf{D}}_o$, and the magnitudes of the index of refractions, n_o and $n_e(\theta)$.

Solving for the index of refraction To determine the directions of the two polarized eigenmodes as well as their indices of refraction for a given direction of propagation, the index ellipsoid can be utilized. For a uniaxial crystal, the equation of the index ellipsoid is given by:

$$\frac{x^2+y^2}{n_o^2}+\frac{z^2}{n_e^2}=1$$

where n_o is the ordinary index and n_e is the extraordinary index. This is an ellipsoid of revolution with the circular symmetry axis parallel to z, as shown in Figure1(a). The direction of propagation is along $\hat{k}$ at an angle θ to the optic (z) axis. Because of the circular symmetry about the z axis, we can choose the y axis to coincide with the projection of $\hat{k}$ on the $x-y$ plane without any loss of generality. The intersection ellipse of the plane normal to $\hat{k}$ with the ellipsoid is shaded in the figure. The two polarization eigen states of $\vec{\mathbf{D}}$ are parallel to the major and minor axes of the ellipse, which correspond to the segments OR and OE. They are perpendicular to $\hat{k}$ as well as to each other. The wave which is polarized along OE is called the extraordinary wave. The index of refraction is given by the length of OE. It can be determined using Figure 1(b), which shows the intersection of the index ellipsoid with the y-z plane.

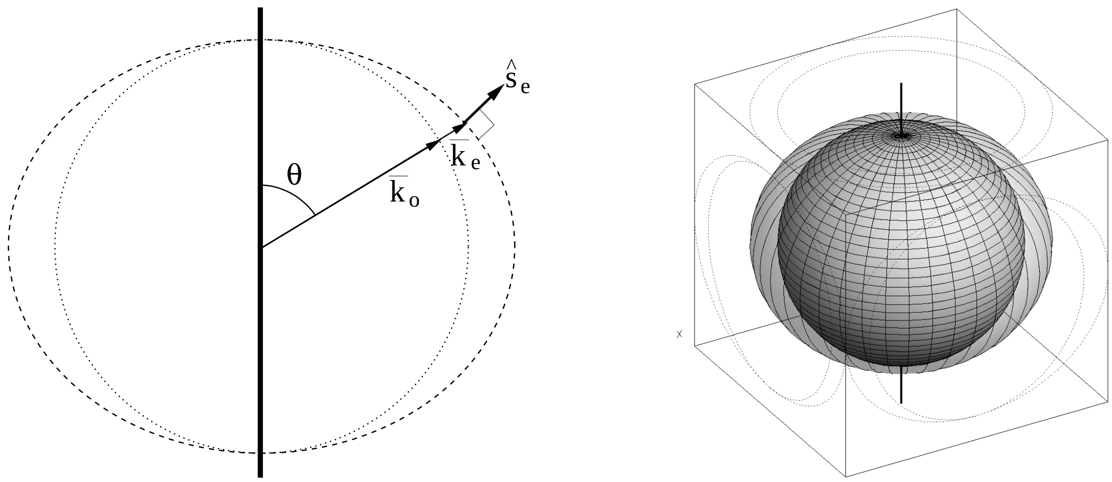


Figure 2: Anisotropic k-space for a uniaxial crystal, showing the directions of the wavevectors, the corresponding polarizations, and the direction of the Poynting vector giving the power flow.

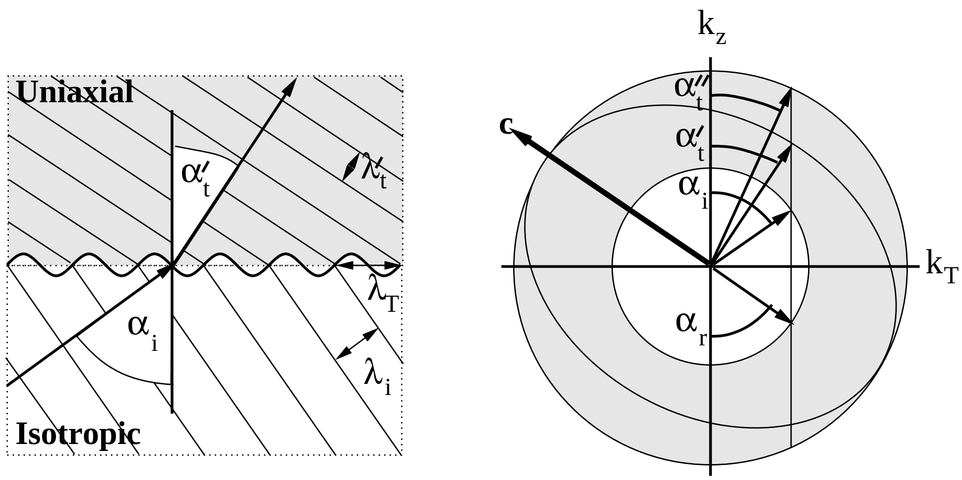
Solving for the index of refraction as a function of direction of propagation yield two eigenvalues for each direction of propagation, each associated with a transverse polarization eigenvector. For a uniaxial crystal the ordinary index n_o is independent The angularly dependent index of refraction is

$$\begin{aligned} n_o(\theta,\phi) &= n_o \\ n_e(\theta,\phi) &= \left[\frac{\cos^2\theta}{n_o^2}+\frac{\sin^2\theta}{n_e^2}\right]^{-\frac{1}{2}} \end{aligned}$$

The concept of momentum conservation (phase matching) at the crystal boundary is very useful for finding the propagation direction(s) of the refracted wave in the anisotropic crystal. Boundary conditions require that the projection of the propagation vectors along the boundary plane must be equal for both the incident and refracted waves. This results in double refraction of a wave incident on the surface of an anisotropic crystal as shown in Figure3. We can therefore write

$$k_0\sin\theta_0 = k_o\sin\theta_o = k_e\sin\theta_e$$

The direction of the energy flow, however, is not always along the propagation direction of the phase of the plane wave, but it is normal to the wave-vector (or index) surface. Using this information, one can find the directions of the refracted beams in a given anisotropic crystal.



a) Real space   b) k space

Figure 3: Refraction from an isotropic medium into a uniaxial anisotropic medium with tilted optical axis with respect to the face normal. The periodicity of the wave crashing into the surface must be conserved and this is shown in a) real space and b) reciprocal Fourier space (also known as k-space or momentum space).

2.2 State of Polarization

Two important mathematical representations of polarized light are the Jones vector for coherent fields and the Stokes vector for general fields.

2.2.1 Jones Calculus

Jones vectors are a relatively compact 2-dimensional complex vector description of the state of polarization of a plane wave, but they describe the phase and amplitude of the electric field of a purely monochromatic wave, and thus can not capture the statistical properties of polychromatic or partially polarized light. The complex Jones vector is $[E_x e^{i\delta_x}, E_y e^{i\delta_y}]^T$, and when normalized to $I = \vec{\mathbf{E}}^\dagger \cdot \vec{\mathbf{E}} = 1$ and referenced to the phase of the x component, simple forms include horizontal=$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, vertical=$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ 45° linear=$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, -45° linear=$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$, RHC=$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$, LHC=$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$, and general with orientation α and phase δ is given by $\begin{bmatrix} \cos\alpha \\ e^{i\delta}\sin\alpha \end{bmatrix}$. Jones matrices represent the transformation of the state-of-polarization (SOP) as 2 by 2 linear operators:

$$\begin{bmatrix} E'_x \\ E'_y \end{bmatrix} = \begin{pmatrix} j_{xx} & j_{xy} \\ j_{yx} & j_{yy} \end{pmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

where the elements can be complex to represent a phase shift. To represent a rotated component in the laboratory frame, rotation operators are used $\underline{\underline{J(\theta)}} = \underline{\underline{\mathcal{R}}}(-\theta)\,\underline{\underline{J}}\,\underline{\underline{\mathcal{R}}}(\theta)$. An x-polarizer Jones matrix is given by $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ while a half waveplate is $\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$ and a quarter

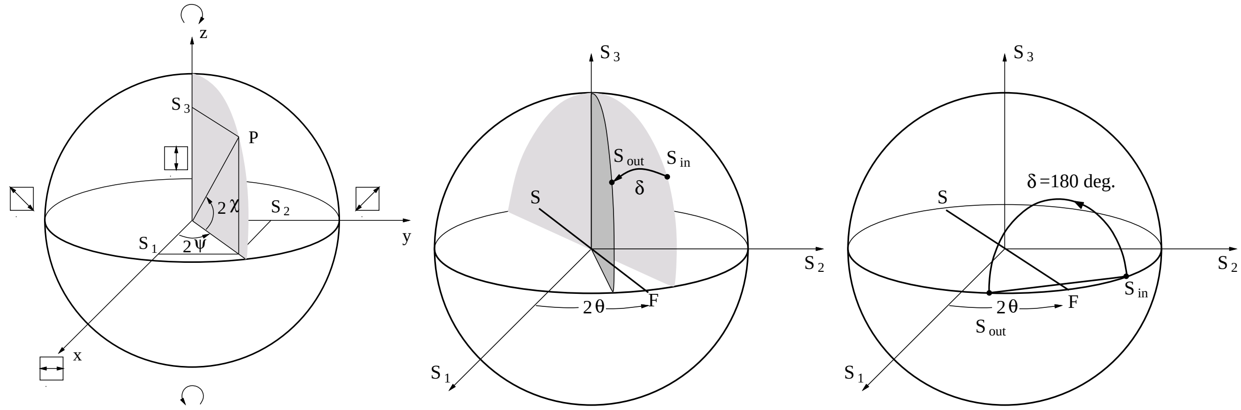


Figure 4: a) Geometry of the Poincare sphere with the linear polarizations on the equator, circular polarizations at the poles, and elliptical polarizations in between. b) Operation of an arbitrary waveplate on an arbitrary polarization as a rigid body rotation of the Poincare sphere. c) The operation of a half waveplate on a linear polarization as a 180 degree rotation of the SOP on the Poincare sphere.

waveplate with the fast axis vertical is $e^{i\pi/4}\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$ A train of components is represented by a series of Jones matrices operating on the Jones vector in the sequence that the beam encounters the components, $\underline{\underline{J}} = \underline{\underline{J_3}}\,\underline{\underline{J_2}}\,\underline{\underline{J_1}}\vec{E}$. A mirror flips the handedness of the coordinate system so must be represented by a special matrix $\underline{\underline{M}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and components after the mirror have the sign of their off diagonal components reversed $\begin{pmatrix} j_{xx} & -j_{xy} \\ -j_{yx} & j_{yy} \end{pmatrix}$

2.2.2 Stokes Calculus and the Poincare Sphere

The Stokes vector can be written as $[S_0, S_1, S_2, S_3]^T$. S_0 is the intensity. S_1 indicates a tendency for the polarization to resemble either a horizontal state ($S_1 = +1$) or vertical state($S_1 = -1$), and is measured as the difference in transmission between horizontal and vertical polarizers. S_2 indicates a tendency for the light to resemble a +45($S_2 = +1$) or -45($S_2 = +1$) polarization, and is measured as the difference in transmission between +45 and -45 polarizers. S_3 describes a tendency of the beam toward right-handedness or left-handedness, reaching $S_3 = +1$ for right hand circular (RHC) and $S_3 = -1$ for left hand circular (LHC), and is measured as the difference in transmission between RHC and LHC polarizers. The geometrical representation of the Stokes parameters as the state of polarization is the Poincare sphere of radius S_0, with the other three parameters forming three orthogonal axes intersecting at the center of the sphere as shown in Fig.4. For describing pure polarization transformations, often the Stokes vector components are normalized by S_0 yielding new components $[1, S_1/S_0, S_2/S_0, S_3/S_0]^T = [1, S_1, S_2, S_3]^T$, since the radius doesnt change as only the polarization is manipulated. The 4 x 4 Mueller matrices represent the transformation of Stokes parameters in an optical system, and obey a similar calculus to Jones, but involving 4 by 4 real matrices describing observable quantities instead of 2 by 2 complex matrices describing field amplitudes. They can be thought of as corresponding to rotation and other motions on the Poincare sphere. The Stokes parameters are in terms of intensity and thus are capable of representing states of unpolarized and partially polarized light.

3 Preparation and prelab

Read the section on crystal optics in your favorite optics book.

- Fowles, Modern Optics, Chap 6.7-8
- Born and Wolf, Principles of Optics, Chap 14
- Shubinikov, Principles of Optical Crystallography
- Yariv and Yeh, Optical Waves in Crystals, chap 3,4,5
- E. Hecht, Optics, Chap 8
- H. J. Juretschke, Crystal Physics, Chap 9

3.1 Prelab

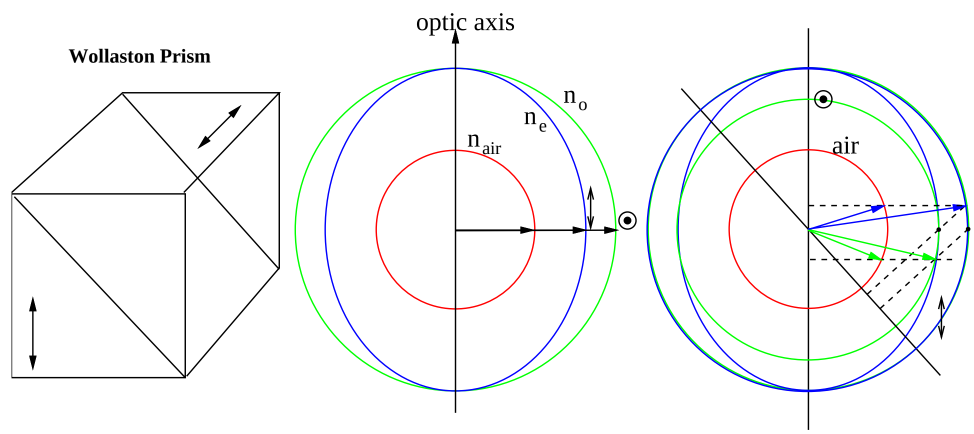


Figure 5: Wollaston prism and the interpretation of its operation in k-space

1. Describe and analyze the operation of a wollaston prism, which is made from two cemented prisms of uniaxial calcite, with a length to aperture L/A ratio of 3. At a wavelength of 632.8 nm the ordinary index of calcite is $n_o = 1.65566$ and the extraordinary index is $n_e = 1.48518$. What is the angular deviation between the two polarizations in this prism? Represent the wave vectors in momentum space. See Fig 5.

2. Consider a crystallographically cut 1cm cube of negative uniaxial calcite illuminated with a tightly focused spherical wave of monochromatic vertically polarized light with $\lambda = .6328\mu$m, with the optical axis of the crystal parallel to the lens axis. Sketch the state of polarization of the input as the optical $\vec{k}$ vector is rotated in a cone about the optical axis, and the decomposition into ordinary and extraordinary components, and sketch the state of polarization at the output of the crystal. When the transmitted light is passed through a horizontal analyzer a set of circular fringes appears, modulated by a cross. Explain using your sketch of the state of polarization at the output of the crystal. How can this effect be used to determine the optical axis of a uniaxial birefringent crystal. See Fig 6.

3. Design an experimental arrangement of waveplates that will produce an arbitrary state of polarization, characterized by an ellipticity b/a, where a is the major axis and b is the minor axis, and an orientation angle Φ of the major axis clockwise from the vertical.

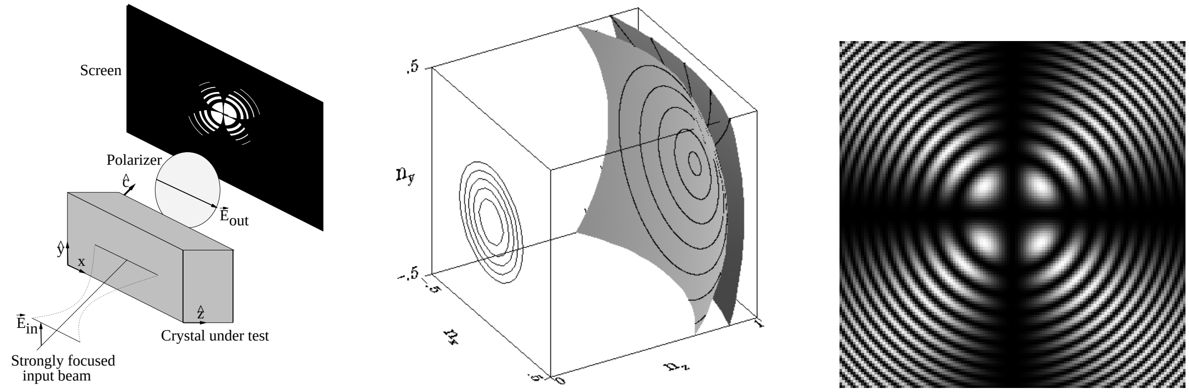


Figure 6: Conoscopic Experimental setup, k-space interpretation, and resulting fringe pattern.

4. See Part One of Procedure. Derive an expression for the transmitted light power.

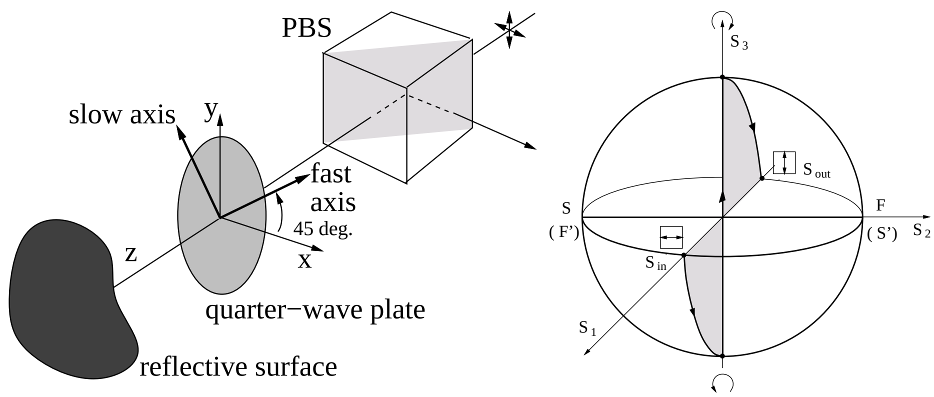


Figure 7: Optoisolator geometry and description on the Poincare sphere.

5. [Not required] Consider a system consisting of a perfect horizontal transmission polarizer, followed by a nominal quarter waveplate with its fast axis oriented at +45 degrees, followed by a retroreflecting mirror. a) Assuming that waveplate is exactly quarter wave at the operating wavelength, what is the reflection backwards through the polarizer. Why? b) If the waveplate has a retardance that deviates away from $\delta = 90$ degrees by a small deviation Δ, eg $\delta = 90 + \Delta$, solve approximately for the reflected power as a function of Δ for small Δ.

6. [Not required] A Soleil-Babinet compensator is made from a movable uniaxial crystal wedge, and a fixed crystal wedge cemented to a compensating crystal plate with its optical axis orthogonal to that of the two wedges, and all optical axis orthogonal to the direction of propagation. The movable wedge is optically contacted to the fixed wedge with index matching oil so that as it is slid back and forth, a variable thickness anisotropic plate is synthesized. Show that when the two plates are of equal thickness a zero retardance waveplate is produced. When the wedge angle is α and the ordinary and extraordinary indices are n_o and n_e at a wavelength λ, at what displacement from this zero retardance position of the movable wedge does the compensator become a half wave plate? What is the advantage of this compensator compared to the Babinet compensator, which does not have the compensating plate, and where the two wedges have their optic axes orthogonal to each other.

3.2 Materials and Equipment

- 1 $LiNbO_3$ crystal
- 1 He-Ne laser
- 1 Calcite ($CaCO_3$) crystal
- 1 Spatial filter
- 2 Crystal mounts
- 2 Irises
- 1 l/4 wave plate (633 nm)
- 1 Collimating lens
- 1 l/2 wave plate (633 nm)
- 1 Microscope objective
- 3 Polarizers
- 1 Mirror
- 1 Polarizing Beamsplitter
- 1 Power meter
- 1 Oscilloscope
- 1 Optical detector

4 Procedure

1. Spatial filter a HeNe laser beam, collimate with a lens, and insert an iris. Then illuminate a pair of crossed polarizers with the HeNe laser beam. What is the residual transmission in the crossed state, and what is the transmission when the two polarizers are aligned? What is the extinction ratio of these polarizers at the HeNe wavelength. With the two polarizers in the crossed state, insert another polarizer between them and plot the transmission of the system as you rotate the central polarizer. How do your results compare with the results of prelab 4?

2. Obtain a quarter wave plate, and determine the optical axes. Describe your procedure for determining the axes. Measure the maximum and minimum transmission through an analyzer as the waveplate is rotated by 10 degree increments. Plot your results, and explain.

3. Variable birefringence retarders can be implemented by tilting a birefringent plate about an axis perpendicular to the optical beam and thereby varying the thickness of the path through the crystal. Align the input polarization at 45° to vertical, and orient a half wave plate with its principal axes horizontal and vertical. Measure and plot the ellipticity of the emerging wave that has passed through the half wave plate oriented with the optic axes at 45 degrees to the incoming polarization as you tilt the wave plate by 15 degree increments. The ellipticity is most simply measured by rotating a polarizer in the output and measuring the maximum and minimum transmission, the ratio giving the a^2/b^2 ratio of major to minor axes.

4. Place a polarizing beamsplitter in the laser beam, and determine the polarization purity of the reflected and transmitted beams, eg for pure V input measure R_V and T_V and for pure H input measure R_H and T_H, and determine the contrasts R_H/R_V and T_V/T_H. Now follow the PBS with a quarter wave plate, and a mirror which retroreflects the light back towards the laser. Measure the light bouncing off the beamsplitter as you rotate the waveplate. Plot your results. Explain how to use this setup as an optoisolator that blocks reflections from going back into the laser cavity thereby disrupting the modal quality. What is the isolation level of your setup, and what limits this isolation.

5. Using your results from Prelab problem 3, set up a quarter wave plate and half wave plate in order to produce light of ellipticity of .25, and an orientation of 45 degrees

away from the vertical. Verify that this is the state of polarization that you obtained. Describe your procedure that demonstrates this.

6. Illuminate a Wollaston prism with the HeNe laser. Describe its operation. Measure the angles of refraction and the polarization of the refracted beams. Now reverse the prism and measure the angles and polarizations of the refracted beams. Are they the same or different? Why?

   Now tilt the prism and measure the field angle over which it operates properly in the horizontal and vertical planes. Now place a second Wollaston prism in one of the deflected beams and measure the isolation ratio of 2 crossed Wollaston prisms. Rotate the second Wollaston by 45°. Why does it produce 2 new refracted beams when we know that the incident state of polarization is pure?

7. Remove wave plates and beamsplitter. Set up a conoscopic system to determine the z-axis of a piece of $LiNbO_3$. Illuminate the crystal with a tightly focused beam. Try to use a short focal length lens (objective lens, even). The shorter the focal length of the lens, the more fringes. Observe the refracted output on the screen or wall as you rotate the crystal. Explain. Place a polarizer on the output and rotate the crystal so that a set of concentric bright and dark circles appears, modulated by a cross. Rotate the crystal to center this pattern in the beam. Rotate the polarizer to give the pattern with the best contrast ratio. Sketch this pattern in your lab book. Rotate the analyzer by 90 and sketch the new pattern. Does a contrast reversal occur as you rotate the output polarizer (analyzer)? Measure the spacing between the successive fringes at some plane a known distance away from the lens focus, and use this measurement to determine the birefringence of the crystal.

8. Rotate the crystal about the vertical axis by 90 degrees and look for a similar polarization interference pattern. What can you see? Try magnifying the fringes using a lens. Rotate the first polarizer such that the incident beam is 45 polarized, and rotate the analyzer. Describe and explain the fringes that you see. Sketch the interference pattern.

9. [OPTIONAL] Illuminate a calcite rhomb with the HeNe laser and observe the beam as it propagates through the crystal. Rotate the state of polarization of the beam and describe your observations. Explain this phenomena. Can you find the optical axis with this technique? Why or why not?