Generation of Nondiffracting Bessel Beams Using a Spatial Light Modulator

Narupon Chattrapiban[1,3], Elizabeth A. Rogers[1,2], David Cofield[1], Wendell T. Hill, III[1,3], and Rajarshi Roy[1,2,3]
[1] *Department of Physics, University of Maryland, College Park, Maryland, 20742*
[2] *IREAP, University of Maryland, College Park, Maryland 20742 and*
[3] *IPST, University of Maryland, College Park, Maryland 20742*
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A laser beam with phase singularities is an interesting object to study in optics and may have important applications in guiding atoms and molecules. Here we explore the characteristics of a singularity in a nondiffracting Bessel beam experimentally, using a programmable spatial light modulator with 64 level phase holograms. The diffraction efficiency using 64 level phase holograms is greatly improved over that obtained using a binary grating. The experiments show that the size and deflection angle of the beam can be controlled in real time. The observations are in agreement with scalar diffraction theory.

In large measure, the role neutral atoms will play in quantum information technology depends on the development of atom-optical elements that are robust and flexible. The atom waveguide is one of the fundamental elements upon which more complex components will be built. Carefully crafted hollow laser beams [1], for example, make ideal conduits for guiding neutral atoms because they can coexist with a variety of atom traps and can be turned on and off quickly. At the same time, long transit times are possible since the atoms spend most of their time in the dark. Networks many meters in length might be possible to construct from waveguides formed by nondiffracting hollow laser beams [2, 3], which preserve their shape during propagation and can be easily generated with holograms. In this letter, we will use a parallel aligned nematic liquid crystal spatial light modulator (SLM) to generate holographic phase gratings. Up to 256 levels of phase shift are possible, which allow more efficient and robust beam generation than is possible with binary phase or amplitude gratings. Furthermore, the SLM provides a flexible way to create a variable grating and generate nondiffracting beams that can be changed and deflected in real-time.

Durnin derived a general exact solution of the scalar wave equation for nondiffracting beams of the form,

$$E(x', y', z' \geq 0, t) = exp[i(\beta z' - \omega t)] \times \int_0^{2\pi} A(\phi) exp[i\alpha(x' \cos\phi + y' \sin\phi)] d\phi \quad (1)$$

where $A(\phi)$ is a complex amplitude function, (ρ, ϕ) are polar coordinates in the grating plane, (x', y') are coordinates in the image plane, z' is the distance between the grating and the image plane, and the parameters α, β and k are related by $\alpha^2 + \beta^2 = k^2$ where k is the wave number [2]. Durnin's equation assumed an infinite aperture, which is not realizable in experiments. We assume, as shown by Vasara, *et al.*, that we have a hologram of radius R characterized by the complex amplitude function $t(\rho, \phi) = A(\phi)e^{i(2\pi\rho/\rho_0)}$ when $\rho \leq R$ and $t(\rho, \phi) = 0$ when $\rho > R$, where $A(\phi)$ is the same complex function as in Eq.(1), and the parameter

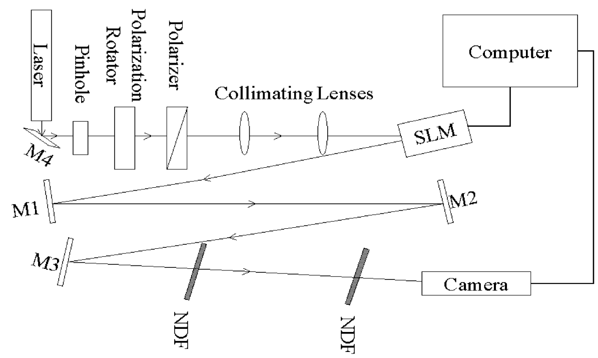

FIG. 1: Experimental setup; The reference beam is reflected off of the SLM and then off three mirrors (M1-M3) in order to increase the propagation length. It then passes through at least one neutral density filter(NDF)before being recorded by a Pulnix TM-72EX CCD camera.

$\alpha = 2\pi/\rho_0$ in Eq.(1), as shown by Vasara, *et al.* [3]. If we choose $A(\phi) = e^{in\phi}$, Eq.(1) becomes a Bessel function of the first kind of order n. In this case $t(\rho, \phi)$ becomes a phase function of the form $t(\rho, \phi) = e^{i\Psi(\rho,\phi)}$, where [3]

$$\Psi(\rho, \phi) = n\phi + 2\pi\rho/\rho_0. \quad (2)$$

The function $t(\rho, \phi)$ is a transfer function in the Fresnel-Kirchoff diffraction integral. The integral yields a function that is proportional to $E(x', y', z' \geq 0, t)$ in Eq.(1) with a square-root dependence on z [3].

In our experiments, the input beam, a linearly polarized TEM_{00} 633nm helium neon laser, is incident on a Hamamatsu SLM through the experimental setup outlined in Fig. 1. We generate the desired phase mask on a computer and the SLM takes the value of the red part of the RGB output of the VGA port of the computer. The VGA output from the computer can send 256 different voltage levels for each color; therefore, the SLM can apply 256 different phase shifts to the incoming light. The SLM also has a bias control to adjust how

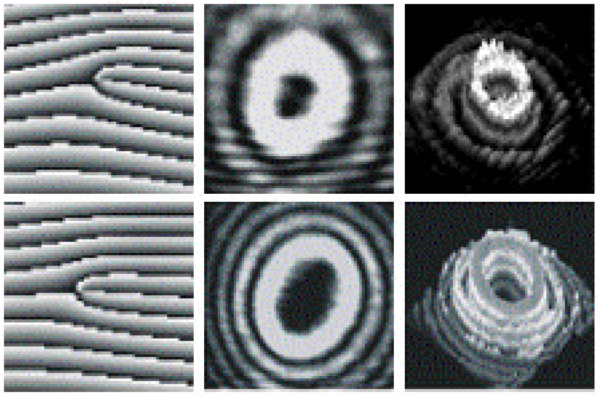

FIG. 2: The phase masks used to produce two different sizes of the dark core of a Bessel beam using a Bessel function of the first kind of order $n = 2$. The first column consists of the actual phase masks that are generated on the computer and placed on the SLM. The parameter is $\rho_0 = 1.4$ mm for the top, notice the larger amount of deviation from horizontal of the fork at the center of the mask due to the smaller magnitude of ρ_0. This results in a smaller hole. The parameter for the bottom is $\rho_0 = 3.2$ mm resulting in a smaller deflection from horizontal for the singularity of the grating, causing a larger hole. The middle column is a 2-dimensional picture of the Bessel beam generated from the gratings to their left, where the intensity is represented by brightness. The right most column is a 3-dimensional image of the hollow beam, where both brightness and height represent intensity.

much of a phase shift each gray scale level induces. The bias voltage ranges from 0 to 5 V, corresponding to phase shifts ranging from 0 to approximately 4π.

The SLM, with an effective area of 2.5×2.5 cm^2, uses the VGA output to make a grayscale pattern on an internal liquid crystal display (LCD). An internal laser propagates through the LCD onto an array of photoelectrodes, which place voltages across a liquid crystal that creates the phase grating. In a parallel aligned liquid crystal, the molecules are arranged in the (x, y) plane and aligned parallel to the y direction, in a lattice. Therefore, in this case, the *director* (the axis of symmetry of the liquid crystal molecules) is along the y-axis, while the reference laser propagates along the z-axis. As a voltage is applied across the liquid crystal molecule, the director reorients off of the y-axis, but remains in the (y, z) plane. The amount of director reorientation is proportional to the applied voltage. When a molecule in the liquid crystal reorients it changes the index of refraction in that region of the liquid crystal, thus retarding the phase of that portion of the beam [4].

If we look back at Eq.(2), we can see that the phase profile resembles that of an axicon, i.e. the phase is proportional to the radius that originated from a singularity (a cone-shaped phase front). The parameter ρ_0 controls the size of the dark core of the Bessel beam. As the value of ρ_0 increases, the size of the dark core increases, as well. The charge or winding number of the singularity is proportional to the amount of angular momentum around the singularity. It is denoted by the charge n [5] of the Bessel beam, which is defined by a closed counterclockwise contour on the phase mask and is equal to the net number of phase discontinuities it crosses. We define the sign of the discontinuity to be positive when the phase jumps from 2π to 0 and negative when the phase jumps the other way. By this definition, the charge of the phase masks in Fig. 2 is equal to +2. In this paper we chose to vary ρ_0 since it can be varied more continuously than n.

To control the size of the dark core of the Bessel beam, we produce 50 phase masks by varying ρ_0 from 0.1 mm to 5.0 mm in steps of 0.1 mm. The resolution of each mask is 500×500 pixels for all of our experiments. The physical size of the dark core, corresponding to ρ_0, varies from approximately 1 mm to approximately 3 mm.

We can also vary the deflection angle of the Bessel beam reflecting from the surface of the SLM [6]. This allows us to steer the beam in a desired direction. The parameter specifying the deflection angle is defined by projecting a reference beam at an angle to the surface of the SLM onto the normal to the surface of the SLM. This phase is denoted as

$$\mathbf{k} \cdot \mathbf{r} = \frac{2\pi}{\lambda} \rho \sin\gamma \cos\phi = 2\pi\nu\rho\cos\phi \qquad (3)$$

where $\mathbf{k}$ is the wave number of the reference beam, γ is the angle between the reference beam and the line normal to the surface of the SLM, $\nu = \sin\gamma/\lambda$ is the carrier frequency where $\lambda = 633$ nm, and the $\cos\phi$ term comes from the inner product of $\mathbf{r}$ and $\mathbf{k}$, projected on the plane of the grating. We observed a linear relationship between the deflection angle and ν.

We coded the phase profile to remain between 0 to 2π and rescaled the phase to 64 equally spaced levels. The total phase profile is now

$$\Psi(\rho, \phi) = (\alpha\rho + n\phi + 2\pi\nu\rho\cos\phi)\, mod\, 2\pi \qquad (4)$$

In all experiments described to this point, $\nu = 4.5 mm^{-1}$. The advantage of a 64 level phase grating over a binary phase grating can be seen in Fig. 3. The intensity of the first ring of the Bessel beam for the 64 level grating is almost an order of magnitude higher than the first ring of the Bessel beam using the binary mask. The integrated power in the first ring of the 64 level mask is 12.4 times the integrated power in the first ring of the binary mask.

With the bias voltage we can control the amount of light in the different orders produced by the grating. Our measurements were made at a bias voltage of approximately 1.88 V, corresponding to approximately

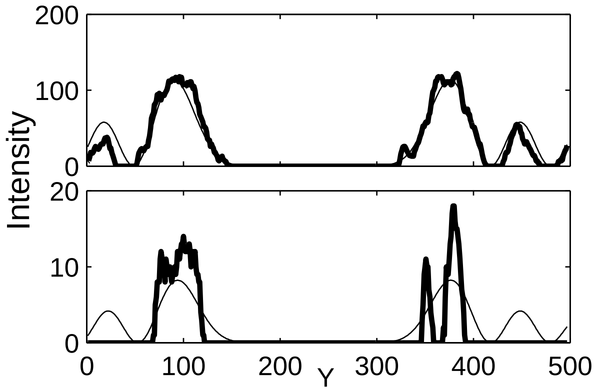

FIG. 3: A comparison between two nonlinear fits of the Bessel beam (J_7) profiles. The top profile was generated from a 64 level phase grating while the bottom profile was generated using a binary phase grating. The ratio of the power in the first ring obtained from the 64 level grating compared to the binary grating is 12.4. The experimental profiles are denoted by the thick line while the Bessel function fit is denoted by the thin line and the amplitudes of both profiles are in similar units. The scale of the y-axis from 0 to 500 is equal to the length of the CCD on the camera used to take the profile. The same neutral density filter was used for both profiles to eliminate the saturation on the CCD camera.

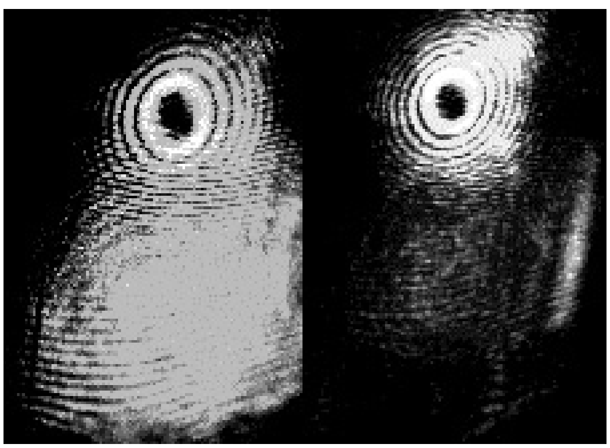

FIG. 4: By adjusting the bias voltage across the liquid crystals, we can control the phase shift produced by the grating. On the left, the bias is not optimized, and we get a significant amount of light in the zero order diffraction beam. This picture was taken at a bias voltage of 1.47 V. On the right, the bias voltage is optimized, at 1.88 V, and almost all of the power is in the first order diffracted beam, while only a small amount remains in the zero order.

a 1.70π phase shift. Fig. 4 is a photograph of the Bessel beam produced by two different levels of bias voltage. At 0 V, only the zero order beam is seen because no phase shift is being imparted onto the read laser beam. Between 0 and 1.88 V, the power is split between the zero and first order diffracted beams, and as the voltage is increased, more power is transferred into the first order. At 1.88 V the power diffracted into the first order beam reaches a maximum, giving the maximum efficiency. As the voltage is increased further, more orders become apparent, and at 2.26 V, both the first and second order diffraction patterns are clear. If we continue increasing the voltage, the second order fades away and the first order appears alone again at approximately 4.65 V.

As the Bessel beam propagates, the clarity of the hole and the surrounding rings increases with distance. As the distance of propagation increases the amount of laser power in the first ring of the diffraction pattern, increases. Thus, as the beam travels further from the SLM, the singularity in the center becomes better defined. Also, as the beam propagates, the anomalous diffraction patterns diverge away from the center of the first order beam, making the Bessel beam clearer. Because the finite aperture of the beam translates into a z dependence for the intensity, at some point the Bessel beam will diverge. The maximum propagation length of the beam is determined by $z_{max} = \rho_0 R/\lambda$ [3] which, in our case is approximately equal to 30 m using ρ_0=1.4 mm, R=1.25 cm (which is the radius of the hologram), and $\lambda = 633$ nm.

In conclusion, we have shown that a 64 level phase mask can generate a propagating Bessel beam of order n that is robust. Most of the intensity of the beam is contained in the first order ring and the size of the hole and the beam deflection angle can be changed in real time.

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