P-82: Diffraction Limited Resolution and Maximum Contrast for Scanning Displays

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Abstract

This paper discusses the theoretical limits for resolution (number of pixels) and maximum contrast for clipped Gaussian and Uniform profile beams in scanning display systems. The results are applicable to scanning microdisplays (e.g. Virtual Retinal Display™) and scanning projection displays.

1. Introduction

Virtual Retinal Display™ (VRD™) technology is based on scanning a photon beam onto the viewer's retina in a 2-D raster pattern using low power red, green, and blue lasers [1,2,3]. The VRD system electronics, combined with light sources and modulators, creates a modulated multicolor light beam. The beam is fed into a single mode fiber and the Gaussian beam emitted from the fiber is scanned in a 2-D raster pattern using mechanical scanners and creates an image at an intermediate image plane. The image is then relayed onto the viewer's retina using an ocular lens. The VRD offers high spatial resolution, large color gamut, and very high luminance [4].

The main contribution of this paper is to establish upper bounds of diffraction limited resolution and maximum contrast for scanned beam displays. While this work has been done in the context of the VRD system, the analysis is applicable generally to scanned beam displays (e.g., laser scanning microdisplays and laser scanning projection displays). The paper has two parts. The first part discusses the resolution of the scanning display systems and extends the previously published work by the author [5]. New analytical equations are derived to compute the diffraction spot size as a function of beam profile for Gaussian, uniform, and hybrid Gaussian beams (truncated Gaussian beams). In the second part, an approximate analytical solution to the diffraction integral is presented. The approximate solution is used to compute the diffraction limited maximum contrast achievable within a circular or rectangular dark area surrounded by all white pixels.

2. Focal Plane Irradiance and Spot Size

Figure 1 illustrates part of a scanning display system. A converging beam with Gaussian profile is incident on a circular aperture (scanning mirror in the figure) of size D_m. Complex wave amplitude of the Gaussian beam can be expressed as

$$U(r, w_m) = \sqrt{\frac{2}{\pi}}\,\frac{1}{w_m}\,e^{-(r/w_m)^2} \tag{1}$$

where w_m is the beam waist radius and r is the distance from the optical axis, and beam irradiance is $I=|U|^2$. Beam amplitude is chosen such that the average unclipped beam power per pixel or the average pixel power incident at the scanner aperture is unity. Power loss due to beam clipping at the aperture depends on the beam truncation ratio $T= w_m/r_m$. Power loss (P_{loss}) and the average pixel power transferred from the aperture (P_{in}) can be expressed as

$$P_{loss}(T) = e^{-2/T^2}$$
$$P_{in}(T) = 1 - e^{-2/T^2} \tag{2}$$

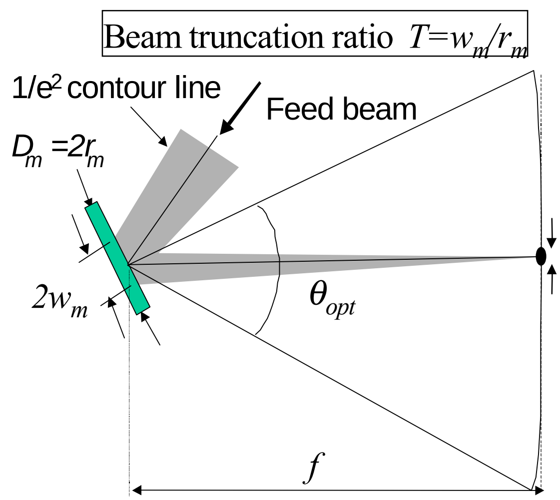

Figure 1: Scanning display geometry cross-section. Only the part from scanner to the intermediate image plane is shown. Total optical scan angle is twice peak-to-peak mechanical scan angle.

The Huygens-Fresnel diffraction integral can be used to calculate the irradiance at the focal plane [6]:

$$I_f(T, r_n) = \frac{2\pi}{(\lambda f_\#)^2}\left[\frac{1}{T}\int_0^1 e^{-(\rho/T)^2} J_o(\pi\rho r_n)\,\rho\,\mathrm{d}\rho\right]^2 \tag{3}$$

where λ is the wavelength, $f_\#=f/D_m$ is the focal-ratio of the focusing geometry, $r_n=r/\lambda f_\#$ is the normalized radial focal plane coordinate, $\rho=r/r_m$ is the normalized aperture coordinate, and J_o is the zeroth order Bessel function of first kind.

Figure 2 illustrates how spot profile, peak irradiance, spot width and energy shifted to the diffraction rings changes with T. For T>>1, the scan mirror is overfilled (i.e., beam profile after the scanner is approximately uniform) and the focused spot profile approximates an Airy pattern, and for T<0.5, clipping at the aperture can be ignored and beam profile remains Gaussian as it propagates.

The diffraction limited focused spot size can be expressed as

$$s = K\lambda f_\# \tag{4}$$

where K incorporates beam clipping effects. Table 1 shows the values of K for full-width at half-maximum irradiance (FWHM) and full-width at 1/e²-irradiance (FWE2) spot sizes. These formulas are

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obtained using curve fitting to numerical solutions of the diffraction integral and are simpler and more accurate (error <1%) compared to other approximate formulas published elsewhere [7].

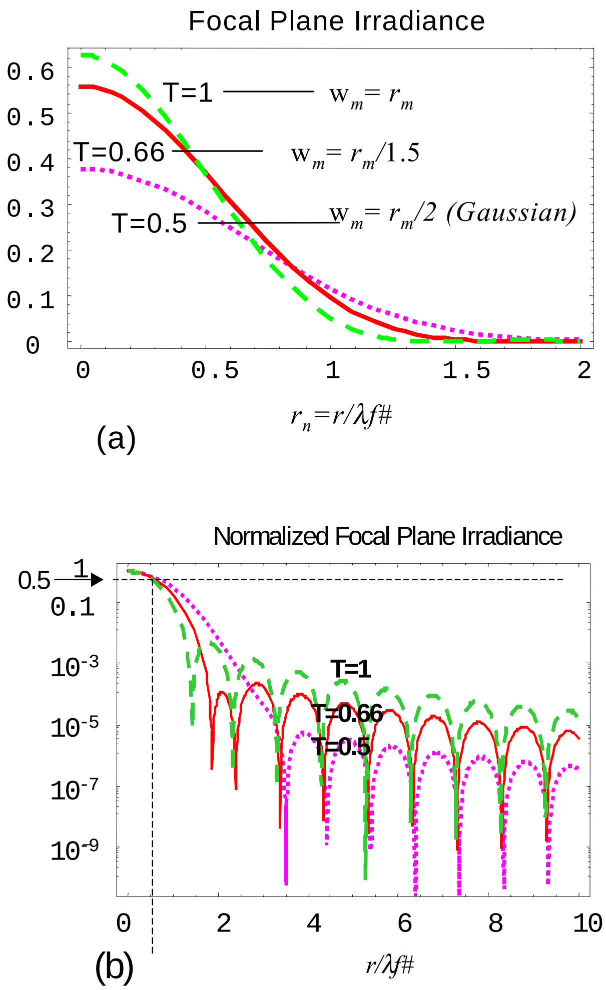

Figure 2: (a) Focal plane irradiance (normalized with $(\lambda f_\#)^2$ for different values of truncation ratio T; (b) Focal plane irradiance in log scale (peak is normalized to unity) illustrating diffraction rings for different T.

Table 1 Spot size formulas for clipped Gaussian beams.

$T = w_m / r_m$ $f_\# = f / D$	Spot Size $s = K \lambda f_\#$
T<0.4 (Gaussian–Negligible clipping)	$K_{FWE-2} = \frac{1.27}{T}$ $K_{FWHM} = \frac{0.75}{T}$
T> 0.4 (Clipped Gaussian) [Error < 1%]	$K_{FWE-2} = 1.654 - \frac{0.105}{T} + \frac{0.28}{T^2}$ $K_{FWHM} = 1.036 - \frac{0.058}{T} + \frac{0.156}{T^2}$

3. Resolution

Maximum resolution or maximum number of pixels for a scanning display system can be expressed as[3,5]

$$N_{\max} = \frac{D\theta_{opt}}{K_{FWHM}\,\lambda} \tag{5}$$

It is assumed that the pixel size is chosen to be equal to the FWHM spot size. More pixels can be fit into the same display area by choosing the pixel size smaller than the FWHM spot size; however, the display system MTF at maximum spatial resolution would become lower than the acceptable limits for high-performance display systems. Note that increasing the truncation ratio from T=0.5 to T=1.0 without changing the $f_\#$ of the focusing geometry results in reducing the FWHM spot size coefficient K_{FWHM} from 1.54 to 1.13, thus improves the scanning system resolution by 27%. Largest value of N is attained for the uniform illumination case ($T \to \infty$ and $K_{fwhm} \to 1.04$). A uniform profile can be achieved either by beam clipping at the expense of beam power or by using efficient optical beam shaping elements that converts a Gaussian beam to a tophat or a uniform beam.

4. Maximum Contrast

An important measure of image quality for displays is the maximum contrast. Consider Figure 3 where a dark circular region that is several pixels wide is surrounded by a white region. The maximum contrast is the ratio of the irradiance attained in the white area (I_{max}) to the minimum irradiance attained at the center of the dark area (I_{min}).

Maximum irradiance can be calculated by summing up properly scaled and shifted irradiance distributions obtained using the diffraction integral. A much easier way to compute I_{max} is to calculate the average irradiance of a white field using the ratio of the average pixel power (P_{in}) to the pixel area (p^2)

$$I_{\max}(p,T) = \frac{1 - e^{-2/T^2}}{p^2} \tag{6}$$

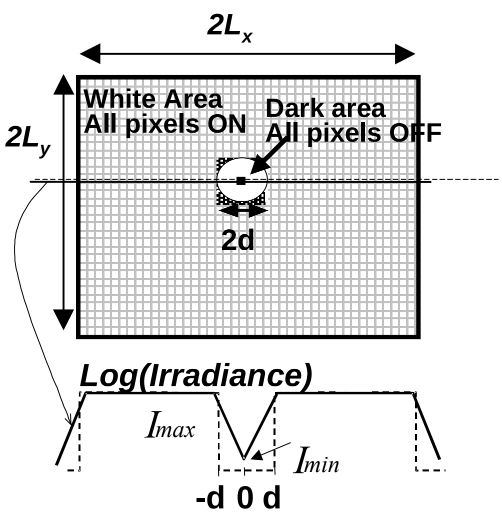

Figure 3: Displayed screen and corresponding irradiance profile in Log coordinates. Gridlines are (imaginary) pixel boundaries for the scanning display system.

Minimum irradiance, I_{min}, is a result of the diffraction spread of the point spread function and can be calculated by summing up the power contributions of all the ON pixels to the pixel at the center of the dark area. This is again a lengthy calculation. One can instead solve the much simpler inverse problem and obtain the same result. Assume only the pixel at the center of the dark area is ON, and compute the total power transferred to all pixels outside a circle of radius d (non-encircled power). The problem of finding the encircled power within the central dark pixel is now reduced to finding the non-encircled beam power within a radius d.

For large arguments, the following approximation can be used for the Bessel function in Eq.3 [8]

$$J_o(x) = \sqrt{2/\pi x}\cos(\pi/4 - x) \tag{7}$$

The approximation results in the following solution for the focal plane irradiance:

$$I_f(T, r_n) = \frac{4e^{-2/T^2} Sin^2 \pi q r_n}{(\lambda f_\#)^2 \pi^3 r_n^3 T^2} \tag{8}$$

where q is a phase factor that is a function of r_n. The approximation is valid for $r_n > 5$ (or $r > 5\lambda f_\#$). The average focal plane irradiance is then given by

$$I_{f,avg}(T, r_n > 5) = \frac{2e^{-2/T^2}}{(\lambda f_\#)^2 \pi^3 r_n^3 T^2} \tag{9}$$

The encircled power, P_e, within the central dark pixel can be calculated by substituting $r_n = r/\lambda f_\#$ and integrating the above equation for $r > d$. The upper bound on r typically is set by the display size, however, for now, assume that the display area is sufficiently large compared to the dark area and carry the limits of integration from d to ∞.

$$\begin{aligned} P_e(T,d) &= \int_d^\infty 2\pi I_{f\_avg}(T,r)\, r\, \mathrm{d}r\, \mathrm{d}\theta \\ &= \frac{4e^{-2/T^2}}{\pi^2 T^2}\frac{\lambda f_\#}{d} \end{aligned} \tag{10}$$

I_{min} is the average irradiance within the pixel area and is given by

$$I_{\min}(T,d,p) = \frac{P_e}{p^2} = \frac{4e^{-2/T^2}}{p^2 \pi^2 T^2 (d/\lambda f_\#)} \tag{11}$$

The maximum contrast is the ratio of the pixel irradiance in the white and dark area and is independent of the pixel size.

$$C_{\max}(T,d) = \frac{I_{\max}}{I_{\min}} = \frac{\pi^2 T^2 (1 - e^{-2/T^2})(d/\lambda f_\#)}{4e^{-2/T^2}} \tag{12}$$

For uniform beams,

$$T \to \infty \text{ and } C_{\max} \to \pi^2 d/\lambda f_\# \tag{13}$$

The analysis can be extended to calculate C_{max} within a rectangular display area of half width d_x and d_y

$$C_{\max}(T,d) = \frac{\pi^3 T^2 (1 - e^{-2/T^2})}{8e^{-2/T^2}(\lambda f_\#)} \frac{d_x d_y}{\sqrt{d_x^2 + d_y^2}} \tag{14}$$

A more general formula that includes the display half width L_x and Ly (defined in Figure 3) can be written using the above formula

$$C_{\max} = \frac{\pi^3 T^2 (1 - e^{-2/T^2})}{8e^{-2/T^2}(\lambda f_\#)} \left(\frac{\sqrt{d_x^2 + d_y^2}}{d_x d_y} - \frac{\sqrt{L_x^2 + L_y^2}}{L_x L_y} \right)^{-1} \tag{15}$$

5. Summary

In this paper we established the upper bounds on resolution and maximum contrast for laser scanning microdisplays and projection displays. The analysis is carried out for circularly symmetrical Gaussian beams that are truncated by the limiting aperture of the scanning system. The results are also valid for uniform beams, which is the limiting case of T→∞.

As illustrated in Figure 4, the resolution is inversely proportional to K_{fwhm}. Increasing T improves the resolution but reduces the maximum contrast. Table 2 summarizes the results of this paper by way of an example.

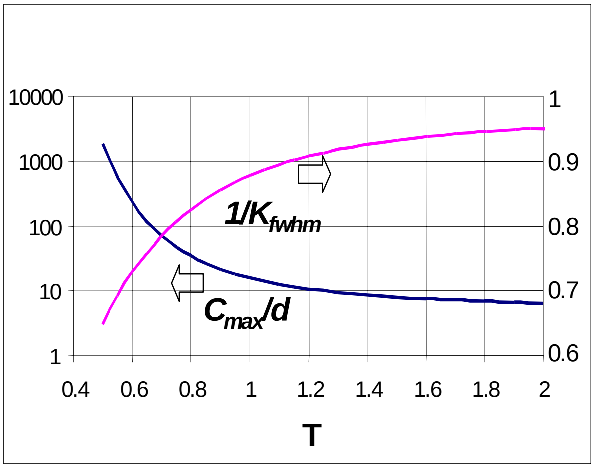

Figure 4: Maximum contrast ratio (normalized with radius of circular dark area d) and reciprocal of K_{fwhm} (proportional to number of resolvable pixels N) as a function of T.

Table 2

Assume a VGA display (640x480) where pixel size is set equal to FWHM spot size, $p=K_{fwhm}\lambda f_{\#}$, and a rectangular dark area that is 64x48 pixels wide at the center is surrounded by all white pixels.

T	$D_x/\lambda f_{\#}$	$d_y/\lambda f_{\#}$	Resolution coefficient (a.u.)	C_{max}
0.5	49.5	37.1	1.0	95293
0.66	41.9	31.4	1.18	4602
1.0	36.4	27.3	1.36	600
∞	33.2	24.9	1.49	171

The optimal choice of beam truncation ratio (T) is in the range 0.5<T<1.0. If beam shaping optics are used to convert the Gaussian laser beam efficiently to a uniform beam, the resolution can be maximized at the expense of maximum contrast. T<0.5 wastes system resolution without any realizable gain in maximum contrast. Note that in addition to the diffraction limitation, the maximum contrast is also limited by the background illumination, modulator dynamic range, and scattered light in the optical system. Maximum contrast ratio for the display system cannot exceed the limit set by any of these factors.

6. Acknowledgements

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7. References

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